SIMULATION OF LOW SPEED FORWARD SHIP MOTION IN THE WIND

Behlül Özdemir

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THESIS

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FORWARD SHIP MOTION IN THE WIND

by

Behlül Özdemir

Thesis Advisor: G. J. Thaler

December 1972

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ABSTRACT

The objective of this study was to examine the relationship between forward ship motion and the effects of aerodynamic and hydrodynamic disturbances on motions in the horizontal plane. It includes course control and stability analysis for the unsteered and steered cases (manual and automatic) using several sets of operating conditions.

Hydrodynamic and aerodynamic effects are considered to be functions of hull motion, rudder (steered cases), propeller and wind effects ratio. A dimensionless mathematical model has been developed and solved with respect to ship axes.

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PHYSICAL QUANTITIES

a_i	Coefficients of perturbation equation in yaw
b_i	Coefficients of perturbation equation in sway
c_i	Coefficients of perturbation equation in surge
$\vec{F}$	Force on the body
J_z	Moment of inertia referred to z-axis
ℓ	Ship length
M	Moment about origin
m	Mass of ship
N	Hydrodynamic and aerodynamic yaw moment
N_a	Aerodynamic yaw moment
$N_{\dot{r}}$	Derivative of hydrodynamic yaw moment with respect to yaw acceleration
N_v	Derivative of hydrodynamic yaw moment with respect to sideslip velocity
N_δ	Derivative of hydrodynamic yaw moment with respect to rudder angle
R	Vector distance of a point from the origin, and the components along the X, Y, and Z axis $\vec{R} = \vec{i}x + \vec{j}y + \vec{k}z$
r	Yaw rate
U	Velocity of the origin $\vec{U} = \vec{i}u + \vec{j}v + \vec{k}w$
u_e	Component of equilibrium ship speed in x-axis direction
u_a	Component of ship velocity relative to air in x-axis direction
U_a	Ship velocity relative to air
U_A	Wind velocity relative to earth

v Component of ship speed in y-axis direction

v_a Component of ship velocity relative to air in y-axis direction

v_e Component of equilibrium ship speed in y-axis direction

X Hydrodynamic and aerodynamic force component in x-axis direction

X_a Aerodynamic force component in x-axis direction

X_p Hydrodynamic force component in x-axis direction due to propeller

$X_{\dot{u}}$ Derivative of hydrodynamic force component in x-axis direction, with respect to surge acceleration

X_{vr} Second derivative of hydrodynamic force component in x-axis direction, with respect to sideslip velocity and yaw angular velocity

X_o Drag coefficient

$X_{\delta\delta}$ Second derivative of hydrodynamic force component in x-axis direction, with respect to rudder angle

Y Hydrodynamic and aerodynamic force component in y-axis direction

Y_a Aerodynamic force component in y-axis direction

Y_r Derivative of hydrodynamic force component in y-axis, with respect to yaw rate

Y_v Derivative of hydrodynamic force component in y-axis direction, with respect to sideslip acceleration

$Y_{\dot{v}}$ Derivative of hydrodynamic force component in y-axis direction, with respect to rudder angle

δ	Rudder angle
δ_e	Equilibrium rudder angle
ρ	Density of water
ρ_a	Density of air
ψ	Heading angle of ship
ψ_a	Direction of wind velocity relative to ship
ψ_A	Direction of wind velocity relative to earth
Ω	Angular velocity of the body about the origin
	$\vec{\Omega} = \vec{i}p + \vec{j}q + \vec{k}r$

DIMENSIONLESS FORMS

Quantities	Typical Symbol	Typical Dimensionless Form
Length	x_r	$x_r' = x_r/\ell$
Force	Y	$Y' = Y/\frac{\rho}{2}\ell^2U^2$
Moment	N	$N' = N/\frac{\rho}{2}\ell^3U^3$
Mass	m	$m' = m/\frac{\rho}{2}\ell^3$
Angular velocity	r	$r' = r\ell/U$
Static force rate	Y_v	$Y_v' = Y_v/\frac{\rho}{2}\ell^2U$
Static moment rate	N_v	$N_v' = N_v/\frac{\rho}{2}\ell^3U$
Rudder force rate	Y_δ	$Y_\delta' = Y_\delta/\frac{\rho}{2}\ell^2U^2$
Damping force rate	Y_r	$Y_r' = Y_r/\frac{\rho}{2}\ell^3U$
Damping moment rate	N_r	$N_r' = N_r/\frac{\rho}{2}\ell^4U$
Inertial coefficient	$Y_{\dot{v}}$	$Y_{\dot{v}}' = Y_{\dot{v}}/\frac{\rho}{2}\ell^3$
Moment of inertia	I_z	$I_z' = I_z/\frac{\rho}{2}\ell^5$
Acceleration	$\dot{u}$	$\dot{u}' = \dot{u}\ell/U^2$
Velocity	u	$u' = u/U$
Angular acceleration	$\dot{r}$	$\dot{r}' = \dot{r}\ell^2/U^2$
Density of air	ρ_a	$\rho_a' = \rho_a/\rho$
Inertial coefficient	$N_{\dot{r}}$	$N_{\dot{r}}' = N_{\dot{r}}/\frac{\rho}{2}\ell^5$

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I. INTRODUCTION

In the last twenty years many mathematical derivations have been performed in the area of motion of vehicles in a fluid medium. Some of these mathematical models have been derived using the six degree of freedom equations of motion.

In this study of six degree freedom equations are reduced to three degrees of freedom with several assumptions. Low ship speed is assumed and coupling effects of roll, pitch and heave motion on to the horizontal plane motion are assumed negligible. It is also assumed that atmospheric motion is uniform.

Aspects of the problem are simultaneous consideration of hydrodynamic and aerodynamic effects as well as nonlinearity in the external disturbances (especially in aerodynamic terms). Equilibrium conditions and perturbation equations are derived from the mathematical model. Nonlinear parts of the mathematical model are linearized with respect to equilibrium conditions.

The effects of wind disturbances on ship motions are studied. A digital computer (IBM model 360/67) was used to solve the transient response in yaw and sway for three different wind directions (head, beam, stern) and constant wind velocities with constant ship speed. Also the eigen value theorem was used to study ship stability. All these computations were made for the unsteered (zero degree rudder deflection) and steered case (with manual as well as automatic control).

When wind velocities are several times the ship speed, even moderate winds create difficulty in controlling the ship course and stability (at low ship speeds). These effects may or may not be controllable, since the hull hydrodynamic reactions are reduced and ship motion is greatly effected by wind force, propeller thrust and coupled rudder force. This situation can cause unusually large hull drift angles extending to between plus or minus 180 degrees. The usual hull hydrodynamic force representation is not applicable beyond 15 to 20 degrees.

II. THE MATHEMATICAL MODEL

A. COORDINATE SYSTEMS AND SHIP MOTION

The coordinate system is defined with respect to the ship itself. It's coordinate origin is chosen to be the center of gravity of the ship as shown in Figure 1.

Positive X axis (lengitudinal axis) is directed to the forward direction along the center line of the ship.

Positive Y axis (transverse axis) is directed to starboard, perpendicular to the plane of symmetry.

Positive Z axis is directed downward, and lies in the vertical symmetry plane, perpendicular to the X-Y plane.

Yaw is a retatienal motion of the ship about the Z axis. (Assume that the ship is moving in the positive X direction and that a moment is applied about the Z axis tending to rotate or turn the ship in clockwise direction as indicated in Figure 2 which shows the positive yaw).

1. Equations of Motion (on the horizontal plane)

The dynamic response in six degrees freedom for the motion of a rigid body [7] is given by equation (1).

$$X = \mathcal{M}[\dot{U}-RV+QW-X_6(R^2+Q^2)+Y_6(PQ-\dot{R})+Z_6(PR+\dot{Q})]$$

$$Y = \mathcal{M}[\dot{V}-PW+RU+X_6(\dot{R}+PW)-Y_6(P^2+R^2)+Z_6(RQ-\dot{P})]$$

$$Z = \mathcal{M}[\dot{W}-QU+PV+X_6(PR-\dot{Q})+Y_6(\dot{P}+QR)-Z_6(Q^2+P^2)]$$

$$L = \dot{P}I_X+(I_Z-I_Y)QR+\mathcal{M}[Y_6(\dot{W}-QV+PV)-Z_6(\dot{V}-PW+RU)]$$

$$M = \dot{Q}I_Y+(I_X-I_Z)PR+\mathcal{M}[Z_6(\dot{U}-RV-QW)-X_6(\dot{W}-QU+PV)]$$

$$N = \dot{R}I_Z+(I_Y-I_X)PQ+\mathcal{M}[X_6(\dot{V}-PW+RU)-Y_6(\dot{U}-RV+QW)] \qquad (1)$$

These equations represent the reaction of the rigid body. They do not include external forces such as rudder force, wind force, aerodynamic and hydrodynamic force of water etc. Applied external hydrodynamic and aerodynamic forces are considered to be functions of hull motion, rudder angle, propeller and wind.

Assuming that roll, pitch, and heave motions may be neglected on the horizontal plane and that x_6, y_6, z_6 are zero for the center of gravity at the origin of the system and noting that on the horizontal plane $\vec{U} = \vec{i}u + \vec{j}v$ (linear velocity), then equation (1) becomes

$$X = m[\dot{U} - RV] \qquad \text{(Surge)}$$

$$Y = m[\dot{V} + RU] \qquad \text{(Sway)}$$

$$Z = 0$$

$$L = 0$$

$$M = 0$$

$$N = \dot{R}I_z \qquad \text{(Yaw)} \qquad (2)$$

These equations are nonlinear. Analytic solution and computer simulation are very difficult. But all nonlinear terms might be linearized by using the Taylor series expansion.

Briefly, small perturbations are considered for:

$$X = m(\dot{u} - rv)$$

$$Y = m(\dot{v} + ru)$$

$$N = \dot{r}I_z \qquad (3)$$

The external force and moments to be included are: [4]

$$X = X(\text{hull}) + X(\text{rudder and prop.}) + X(\text{wind})$$

$$Y = Y(\text{hull}) + Y(\text{rudder and prop.}) + Y(\text{wind})$$

$$N = N(\text{hull}) + N(\text{rudder and prop.}) + N(\text{wind}) \qquad (4)$$

So N, X, Y represent total hydrodynamic and aerodynamic forces and moments which are generated by the ship motion, rudder, propeller and wind. The following assumptions permit elimination of relatively unimportant terms in the hydrodynamic and aerodynamic expressions: [1]

1. Ship asymmetry with respect to the centerline plane is minor (this permits elimination of odd ordered terms for the X force expansion and even ordered terms from the Y and N expansion).

2. All terms higher than third order are unimportant.

3. Rudder force and moment derivatives of higher than first order, and effects of rudder angular rate are negligible.

4. The lengitudinal component of ship speed is much greater than the lateral component (u>>v).

5. Based on experimental and analytical results, nonlinear hydrodynamic terms in the yaw and sway equation for the stability analysis are considered minor, except for certain nonlinear hydrodynamic terms in the surge equations such as X_{vr}'.

6. Atmospheric motion is uniform.

Thus equation (4) may be written as: [1]

$$X = C_1\dot{u}+C_2u^2+C_3vr+C_4u^2\delta^2+x_1v_aU_a+x_p$$

$$Y = B_1\dot{v}+B_2uv+B_3ur+b_yU^2\delta+Y_1v_aU_a$$

$$N = A_1\dot{r}+A_2uv+A_3ur+A_4v^2\delta+N_1u_av_a+N_2v_aU_a \qquad (5)$$

All hydrodynamic and aerodynamic constants are defined in the appendis.

The hydrodynamic effects, forces are defined as a function of the acceleration and velocities of the hull.

The rudder deflection will also produce an additional resistance, an axial force proportional to S^2. This force is expected to be small. The lateral force and moment are proprotional to S. [5]

Forces and moments due to wind acting on the above water protions of the hull act on the ship in the horizontal plane. It is necessary to determine the relative wind speed and direction with respect to the ship in it's own reference frame.

Magnitude and direction of wind is shown on Figure 1. The relative wind speed components along the ship axes are

$$u_a = u+U_a \cos(\psi_a+\psi) \qquad \text{in X direction}$$

$$v_a = v-U_a \sin(\psi_a+\psi) \qquad \text{in Y direction}$$

Then

$$U_a = (u_a^2 + v_a^2)^{\frac{1}{2}}, \quad \psi_a = \tan^{-1}\frac{v_a}{u_a} \qquad (6)$$

U_a and ψ_a are wind velocity and direction along the earth axis.

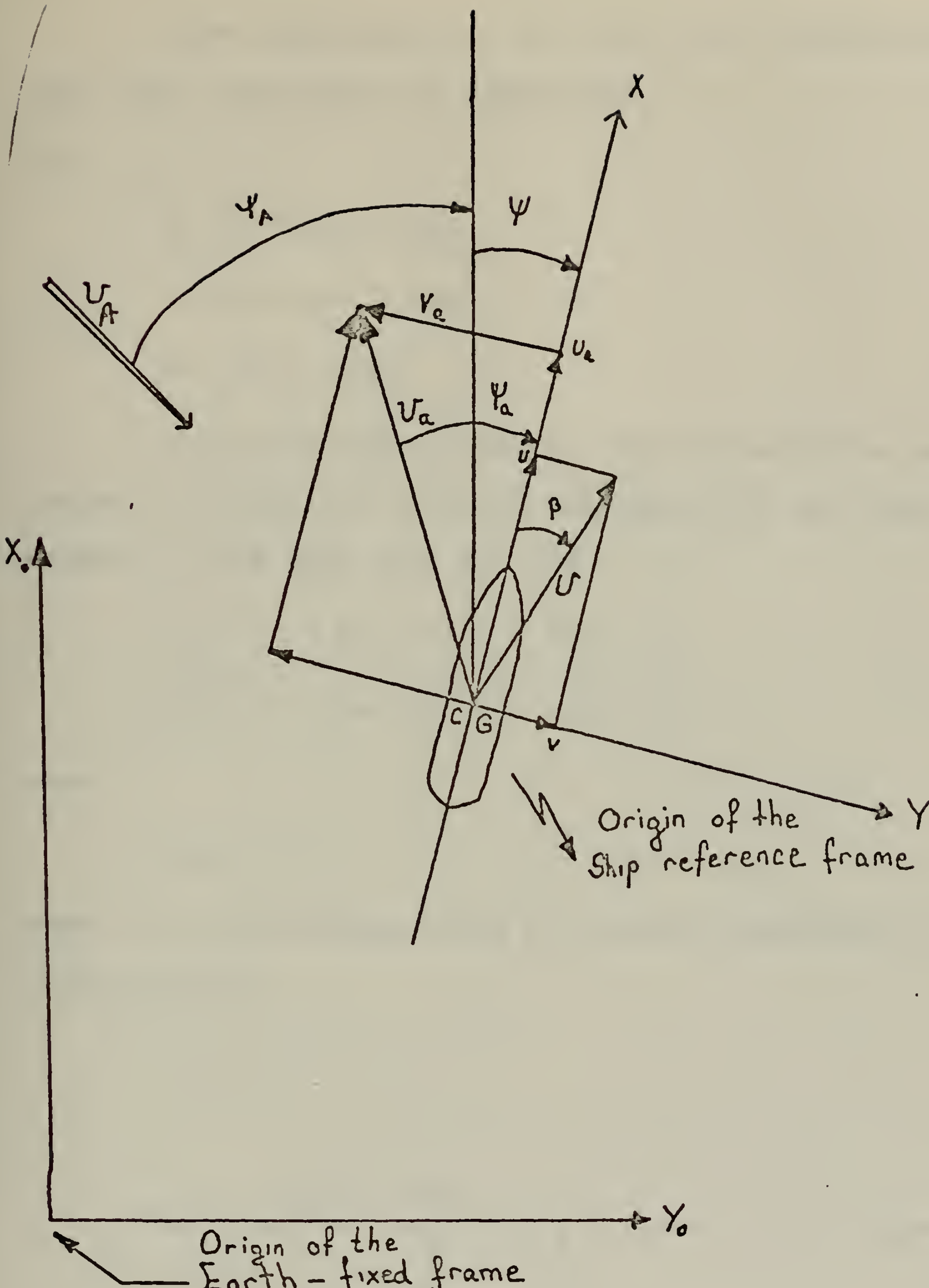


Figure 1. Relation of Wind Velocity and Direction to the Ship Axes.

From equations (3), (4), and (5), accelerations in yaw, sway, and surge are determined:

So

$$X = \dot{u}(m - x_{\dot{u}}) - mrv$$

$$Y = \dot{v}(m - y_{\dot{v}}) + mru$$

$$N = \dot{r}(I_z - N_{\dot{r}}) \tag{7}$$

The relationship between velocities of the ship with respect to the earth axis and velocities of the ship with respect to the body axis is: [3]

$$\dot{Y}_o(t) = u \sin(\psi) + v \cos(\psi)$$

$$\dot{X}_o(t) = u \cos(\psi) - v \sin(\psi)$$

also

$$\dot{\psi}(t) = r \tag{8}$$

Where ψ is the heading angle of the ship relative to the original path.

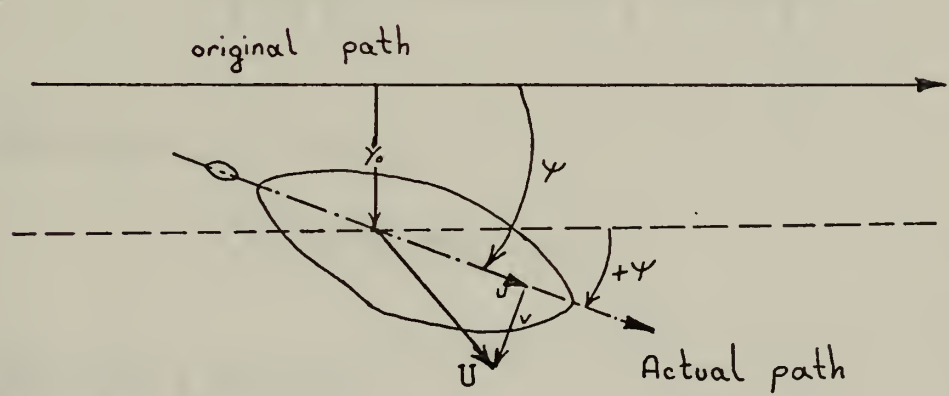


Figure 2. Positive Direction of Yaw.

$$Y_o = \int (u.\sin(\psi) + v.\cos(\psi))\,dt + Y_o(o)$$

$$X_o = \int (u.\cos(\psi) - v\sin(\psi))\,dt + X_o(o) \qquad (9)$$

when ψ is small:

$$\sin(\psi) \simeq \psi\ , \quad \cos(\psi) \simeq 1 \quad U \simeq Ue \text{ (equilibrium case)} \qquad (10)$$

So

$$\dot{Y}_o(t) = Ue \cdot \psi + v$$

$$X_o(t) = Ue - v.\psi \qquad (11)$$

Combining equation (5) and (7) and putting in nondimensional form. [2]

$$\frac{\rho}{2}\ell^3(m'-x\dot{u}')\dot{W} = \frac{\rho}{2}\ell^2[(x_o'+x_{ss}'\delta^2)u^2+\ell(x_{vr}'+m')vr+\rho_a'x_a'U_a^2+x_p'U^2]$$

$$\frac{\rho}{2}\ell^3(m'-y_{\dot{v}}')\dot{v} = \frac{\rho}{2}\ell^2[y_v'uv+\ell(y_r'-m')ur+y'\delta u^2+e_a'y_a'U_a^2]$$

$$\frac{\rho}{2}\ell^5(I_z'-N_{\dot{r}}')\dot{r} = \frac{\rho}{2}\ell^3[N_v'uv+\ell N_r'ur+N'\delta u^2+e_a'N_a'U_a^2 \qquad (12)$$

where

$$\frac{\rho_a}{2}\ell^3U_a^2N_a' = N_a = \frac{\rho}{2}\ell^3U^2N_a'$$

$$N_a' = \rho_a'U_a'^2\left(\frac{N_{a_1}'}{2}\sin(2\psi_a)+\frac{N_{a_2}'}{2}\sin(\psi_a)\right)$$

$$\frac{\rho_a}{2}\ell^2U_a^2y_a' = y_a = \frac{\rho}{2}\ell^2U^2y_a'$$

$$y_a' = \rho_a'U_a'^2Y_{a_1}'\sin(\psi_a)$$

$$\frac{\rho_a}{2}\ell^2U_a^2X_a' = X_a = \frac{\rho}{2}\ell^2U^2X_a'$$

$$X_a' = \rho_a'U_a'^2X_{a_1}'\cos(\psi_a) \qquad (13)$$

$\rho_a' = \rho_a/\rho$, $u \simeq U$ since $u >> v$)

Expanding in Taylor's series around the equilibrium conditions:

$$u = ue \qquad v = ve \qquad \delta = \delta e$$

$$r = \dot{x} = \ddot{x} = \dot{u} = \dot{v} = \quad = 0 \tag{14}$$

Substitute equation (14) into equation (12)

$$N_v' ueve + N_\delta' \delta e\, ue^2 + N_a' ue^2 = 0$$

$$y_v' ueve + y_\delta' \delta e\, ue^2 + y_a'\, ue^2 = 0$$

$$x_o' ue^2 + x_{\delta\delta}' \delta e^2\, ue^2 + x_a'\, ue^2 + x_p' ue^2 = 0 \tag{15}$$

Sideslip velocity and rudder angle in the equilibrium condition can be expressed as: [1]

$$Ve' = \frac{Y_a' N_\delta' - N_a' Y_\delta'}{N_v' Y_\delta' - Y_v' N_\delta'}$$

$$\delta e = \frac{N_a' Y_v' - Y_a' N_v'}{N_v' Y_\delta' - Y_v' N_\delta'} \tag{16}$$

Perturbation terms (small deviation from the initial straight course) may be denoted by bars. So actual motion is

$$ue = ue + \bar{U} \qquad V = ve + \bar{V} \qquad S = Se + \bar{S} \qquad \psi = \bar{\psi}$$

$$\dot{\psi} = \bar{\dot{\psi}} \qquad \dot{u} = \bar{\dot{u}} \qquad \dot{v} = \bar{\dot{v}} \tag{17}$$

Perturbations in aerodynamic quantities due to $\bar{\psi}$, $\bar{v}'$ and $\bar{u}'$ are:

$$\bar{N}'_a = \frac{\partial N'_a}{\partial \psi}\bar{\psi} + \frac{\partial N'_a}{\partial v'}\bar{v}' + \frac{\partial N'_a}{\partial u'}\bar{u}'$$

$$\bar{Y}'_a = \frac{\partial Y'_a}{\partial \psi}\bar{\psi} + \frac{\partial Y'_a}{\partial v'}\bar{v}' + \frac{\partial Y'_a}{\partial u'}\bar{u}'$$

$$\bar{X}'_a = \frac{\partial X'_a}{\partial \psi}\bar{\psi} + \frac{\partial X'_a}{\partial v'}\bar{v}' + \frac{\partial X'_a}{\partial u'}\bar{u}' \qquad (18)$$

Where

$$\frac{\partial N'_a}{\partial \psi} = \rho'_a[(N'_{a_1}\cos 2\psi_a + N'_{a_2}\cos\psi_a)(U'^2_a - u'_a)$$
$$-2U'_a \sin\psi_a (\frac{N'_{a_1}}{2}\sin 2\psi_a + N'_{a_2}\sin\psi_a)]$$

$$\frac{\partial N'_a}{\partial v'} = -\rho'_a U'_a[(N'_{a_1}\cos 2\psi_a + N'_{a_2}\cos\psi_a) + 2(\frac{N'_{a_1}}{2}\sin 2\psi_a$$
$$+ N'_{a_2}\sin\psi_a)\sin\psi_a]$$

$$\frac{\partial N'_a}{\partial u'} = \rho'_a U'_a[(N'_{a_1}\cos 2\psi_a + N'_{a_2}\cos\psi_a)(-\sin\psi_a)$$
$$+ 2(\frac{N'_{a_1}}{2}\sin 2\psi_a + N'_{a_2}\sin\psi_a)\cos\psi_a]$$

$$\frac{\partial Y'_a}{\partial x} = \rho'_a y'_{a_1}[\cos\psi_a(U'^2_a - u'_a) - 2U'_a\sin^2\psi_a]$$

$$\frac{\partial Y'_a}{\partial v'} = -\rho'_a U'_a Y'_{a_1}(1 + \sin^2\psi_a)$$

$$\frac{\partial Y'_a}{\partial u'} = \rho'_a U'_a Y'_{a_1}\cos\psi_a \sin\psi_a$$

$$\frac{\partial X'_a}{\partial x} = -\rho'_a X'_{a_1} \sin\psi_a \, (U'^2_a + u'_a)$$

$$\frac{\partial X'_a}{\partial v'} = -\rho'_a X'_{a_1} U'_a \cos\psi_a \sin\psi_a$$

$$\frac{\partial X'_a}{\partial u'} = \rho'_a X'_{a_1} U'_a (1 + \cos^2\psi_a) \tag{19}$$

The coefficients in equation (19) where taken from reference [2]. Aerodynamic coefficients are shown in Figure 3.

Substitute equation (17) and (18) into equation (12) and obtain the perturbation equation in the following form:

$$(m'-x'_{\dot u})\dot{\bar u}' = (\delta_{vr}'+m')v_e'\bar r' + \frac{2x'_a}{2x}\bar\psi + \frac{2x'_a}{2v'}\bar v' + (2X'_o + 2X'_{\delta\delta}\delta_e^2 + \frac{2X'_a}{2u'})$$

$$\bar u' + 2X_{\delta\delta}'\delta_e\bar\delta$$

$$(m'-y_v')\dot{\bar v}' = (y_v'-m')\bar r' + \frac{2Y'_a}{2x} + (y_v' + \frac{2Y'_a}{2v'})\bar v' + (y'_v v'_e + 2y'_\delta \delta e + \frac{2Y'_a}{2u'})$$

$$\bar u' + y'_\delta\bar\delta$$

$$(I'_z - N'_{\dot r})\dot{\bar r}' = N'_r\bar r' + \frac{2N'_a}{2\psi}\bar\psi + (N'_v + \frac{2N'_a}{2v'})\bar v' + (N'_v v'_e + 2N'_\delta \delta_e + \frac{2N'_a}{2u'})$$

$$\bar u' + N'_\delta\bar\delta \tag{20}$$

The mathematical model which is shown in the above equations is developed only for the forward ship motion. The stern ship motion equations are written by putting negative signs in front of some of the coefficients.

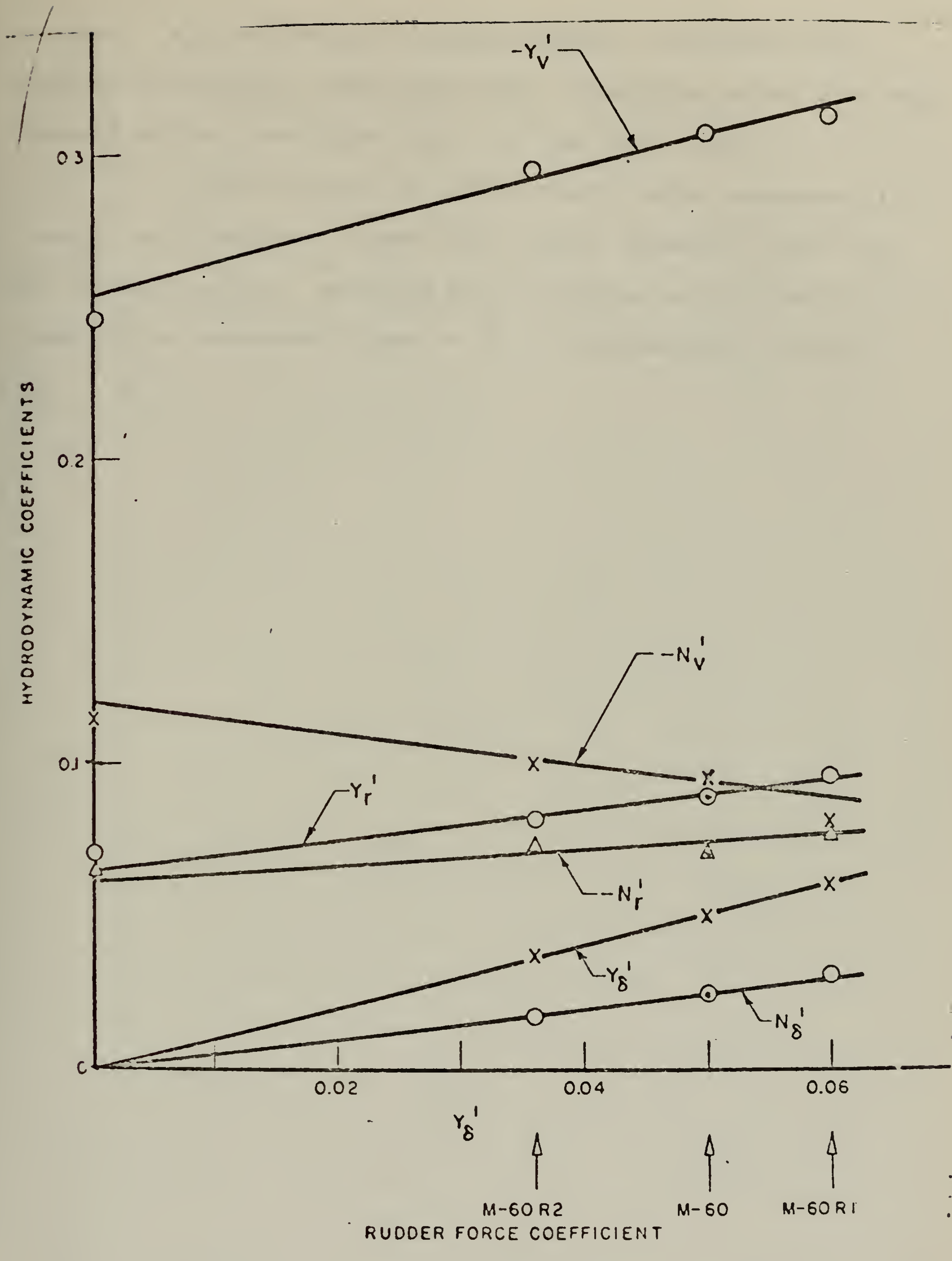


Figure 3. Hydrodynamic Coefficients Vs Rudder Force Coefficient.

Example: N_v (derivative of hydrodynamic yaw moment with respect to sideslip velocity) takes a negative value for the forward motion, positive value for the stern motion.

Y_r (derivative of hydrodynamic force component in Y-axis, with respect to yaw rate) takes negative value for the forward motion, positive for the stern motion but is not a sensitive parameter since m' is the predominant term in $(Y_r' - m')$.

III. CONTROL STUDIES

The linearized equation of motion was derived and found in equation (20). In this part two different problems were simulated. One of them was manually steered ship motion in wind disturbance. The other was automatically steered ship motion under the same effect. So computer program (II) and (III) were developed for these purposes respectively. The computer model is shown in Figure 4, for these programs. Program (II) and (III) are similar to each other. There is only one difference in section 4 of both programs. Program (II) was integrated using the second order equation, however program (III) was integrated with the third order equation. This difference was caused by the feedback loop (page 38). Also the Laplace Transform Technique could have been used to obtain the system description for simplicity and system characteristic equation for the parameter plane program. The characteristic equation was calculated in the computer program. Also three more additional and different mathematical sections were calcualted in the same program. One of these sections included the parameter calculation from equation (6) through equation (20). Secondly the transfer functions of yaw and sway were calculated from the three degree of freedom equation. The last section was the polynomial calculation for the eigen values.

The last program, (parameter plane (IV), was used to get optimal values for yaw gain constant and yaw rate gain constant using the constant zeta (function omega) and constant

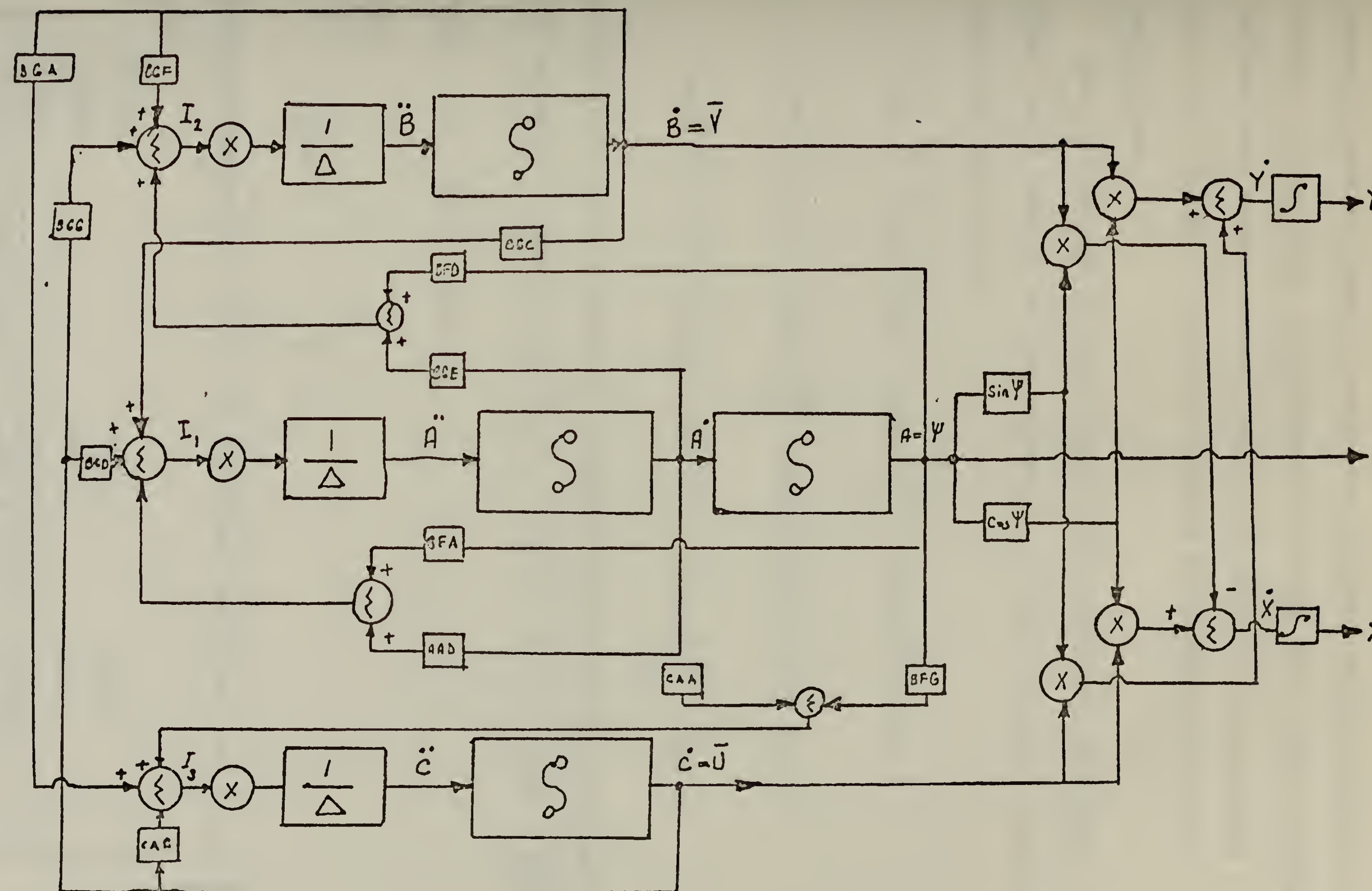


Figure 4. Computer Model for Program II and III.

omega (function zeta) curves for automatically steered ship motion. According to reference [1] and [6], these optimal values were chosen from Figures 40 through 44 assuming zeta = 6, omega = 11.5 for varying wind direction and velocities.

The transform of the linearized equations of motion (20) from the T domain to the S domain, for all initial conditions equal to zero are:

$$
\begin{aligned}
&(ABC.S^2 - AAD.S - BFA)\bar{\psi} - (BGD.S)\frac{\bar{U}}{\delta} - (BGC.S)\frac{\bar{V}}{\delta} = AAE.\bar{\delta} \\
&(-BGE.S - BFD)\bar{\psi} - (BGG.S)\frac{\bar{U}}{\delta} + (ABD.S^2 - BGF.S)\frac{\bar{V}}{\delta} = AAC.\bar{\delta} \qquad (21) \\
&(-CAA.S - BFG)\bar{\psi} + (ABE.S^2 - CAB.S)\frac{U}{\delta} - (BGA.S)\frac{\bar{V}}{\delta} = CAC.\bar{\delta}
\end{aligned}
$$

In equation (21) different letters are used for simplicity. For example: $ABC = I_z' - N_{\dot{r}}'$, $AAD = N_r'$, $BFA = \frac{\partial N_a'}{\partial \psi}$,

$ABE = m' - x_{\dot{u}}'$, etc.

let

$$\frac{\bar{u}}{s} = C \quad , \qquad \frac{\bar{v}}{s} = B$$

Applying Cramer's rule, the transfer function for $\frac{\psi}{\delta}$ and $\frac{B}{\delta}$ are:

$$
\frac{\bar{\psi}}{\bar{\delta}} = \frac{\begin{vmatrix} AAE & -BGD.S & AAE \\ AAC & -BGG.S & AAC \\ CAC & ABE.S^2-CAB.S & CAC \end{vmatrix}}{\begin{vmatrix} (ABC.S^2-AAD.S-BFA) & -BGD.S & -BGC.S \\ (-BGE.S-BFD) & -BGG.S & ABD.S^2-BGF.S \\ (-CAA.S-BFG) & ABE.S^2-CAB.S & -BGA.s \end{vmatrix}} \qquad (22)
$$

The open loop transfer function is:

$$\frac{\psi}{\delta} = \frac{VVA.S^2+VVC.S+VVF}{SSA.S^4+SSB.S^3+SSE.S^2+SBS.S+SES} \qquad (22)$$

Where

VVA = - ABC.ABD.ABE   etc.

All terms were calculated using program 1.

In this case, the results are a second order numerator and fourth order denominator.

Transfer function $\frac{B(s)}{\bar{\delta}(s)}$, was also obtained using Cramer's rule.

$$\frac{B}{\bar{\delta}} = \frac{\begin{vmatrix} ABC.S^2-AAD.S-BFA & -BGD.S & AAE \\ -BGE.S-BFD & -BGG.S & AAC \\ -CAA.S-BFG & ABE.S^2-CAB.S & CAC \end{vmatrix}}{\begin{vmatrix} ABC.S^2-AAD.S-BFA & -BGD.S & -BGC.S \\ -BGE.S-BFD & -BGG.S & ABD.S^2-BGF.S \\ -CAA.S-BFG & ABE.S^2-CAB.S & -BGA.S \end{vmatrix}} \qquad (23)$$

The transfer function between B and $\bar{\delta}$ is:

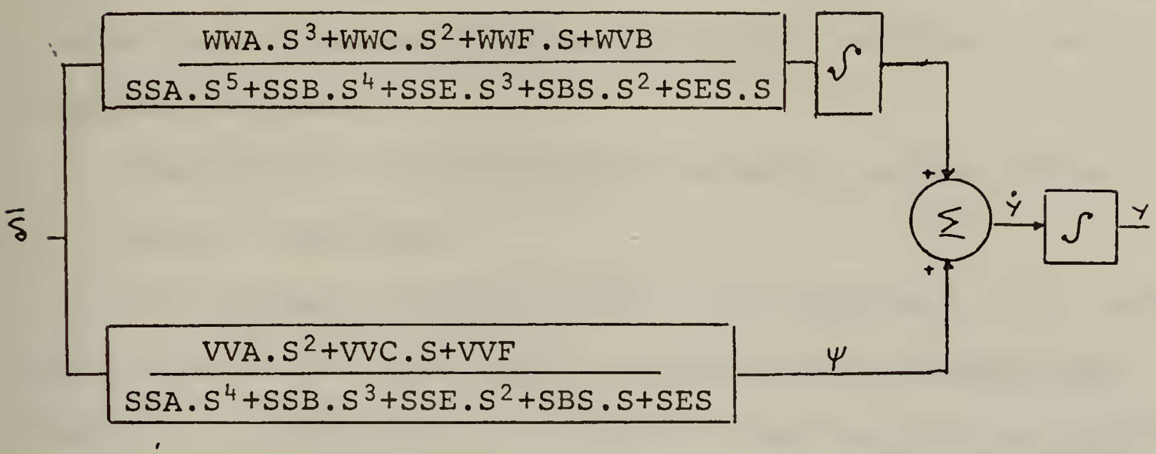
$$\frac{B}{\bar{\delta}} = \frac{WWA.S^3+WWC.S^2+WWF.S+WVB}{SSA.S^5+SSB.S^4+SSE.S^3+SBS.S^2+SES.S} \qquad (24)$$

where

WWA = -ABC.AAC.ABE etc.

Again all terms were calculated using program 1.

For the lateral displacement, sway is defined by



The transfer function is

$$\frac{Y}{\bar{\delta}} = \frac{\mathrm{WWA.S} + (\mathrm{WWC}+\mathrm{VVA})\mathrm{S} + (\mathrm{WWF}+\mathrm{VVC})\mathrm{S}+\mathrm{WVB}+\mathrm{VVF}}{\mathrm{SSA.S} + \mathrm{SSB.S} + (\mathrm{SSE}+\mathrm{SSA})\mathrm{S} + (\mathrm{SBS}+\mathrm{SSB})\mathrm{S} + (\mathrm{SES}+\mathrm{SSE})\mathrm{S} + \mathrm{SBS.S}+\mathrm{SES.S}} \tag{25}$$

A. MANUALLY STEERED SHIP MOTION

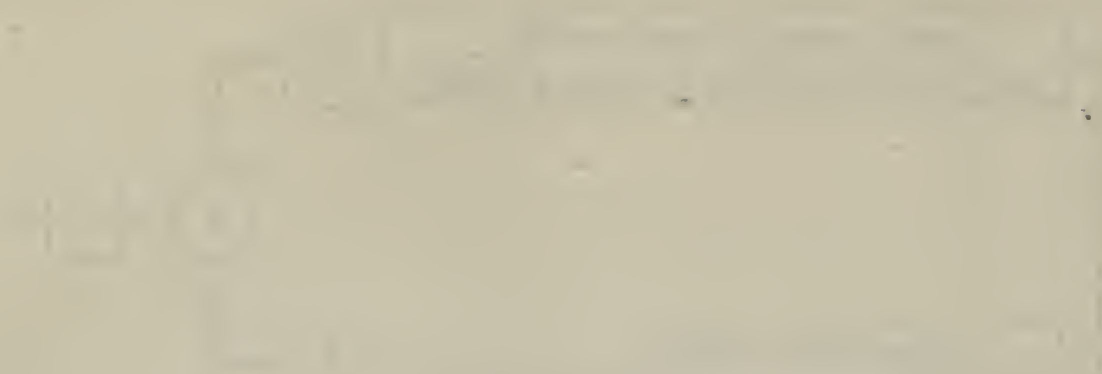
As mentioned before, the goal of this section is to get the ship trajectories in constant wind effect with low ship speed. For this purpose computer program (II) was developed and applied to equation (20). Two situations were considered:

Situation 1: zero degree rudder deflection (unsteered).

Situation 2: positive 15 degree rudder deflection.

All these situation were studied assuming that the ship moved in the forward direction at a 1 mile/hr speed in a calm sea. Each of the wind velocities was applied suddenly.

Computer program II responses are shown in Figures 5 through 30 for yaw and sway. These results were also tabulated in Tables 1 through 4. The first test on the unsteered



situation is shown in the computer outputs from Figure 5 to Figure 24 for three different wind directions and various velocities. Tabulated values are shown on Table 1 and 2, for yaw and sway.

The effects are examined by referring to the tables.

Casel: Head Wind

As is shown in Tables 1 and 2 and Figures 5 - 10, effects of the head wind were small on the straight forward ship motion. In this test, a 5 mile/hr wind velocity is applied to the ship which created -10^{-3} rad. variation on the course and drift was -7.4×10^{-3} ft. in 5 minuted.

Case 2: Beam Wind

Secondly, beam wind disturbances were examined for various velocities. According to Tables 1 and 2 and Figures 11 -16, the ship greatly deflected from a straight course where the wind velocity exceeded t miles/hr. Deviation was positive 22 degree (.395 rad) on the straight course and positive 9.2 ft drift in 5 minutes when the 7 miles wind velocity was applied, as shown in Figures 15 and 16.

Case 3: Stern Wind

In this part, the ship was disturbed with stern wind which created great difficulty on the straight course. Five miles/hr wind velocity created -2.29 degree (-.04 rad) deviation and ∓.28 feet drift in 5 minute. But 7 miles/hr wind velocity causes the ship to deviate by -332.3 degree from the straight course in 4 minute 10 second. Drift was -110 ft in 4 minute but this value was reduced to -15 ft in 5 minute.

Figure	Wind Direction (degree)	Wind Velocity (miles)	Time (sec)	Yaw (rad.)
	0	1	300	-3×10^{-4}
	180	1	300	-14×10^{-4}
	0	3	300	-7.5×10^{-4}
	180	3	{50 300	{-9×10^{-4} -6.5×10^{-4}
	90	3	300	2×10^{-4}
	0	5	{100 300	{-2×10^{-3} -1.4×10^{-3}
	180	5	300	-4×10^{-2}
	90	5	300	8×10^{-3}
	180	7	300	-5.8
	90	7	300	0.395

Table 1. Variation of Yaw for Unsteered Ship.

Figure	Wind Direction (degree)	Wind Velocity (miles)	Time (sec)	Sway (ft)
	0	1	300	-2.9×10^{-3}
	180	1	300	$-.5\times10^{-4}$
	0	3	300	-6.2×10^{-4}
	180	3	{ 80 300	{ 0.0 -4.4×10^{-3}
	90	3	300	2.7×10^{-3}
	0	5	300	-7.4×10^{-3}
	180	5	300	-.28
	90	5	300	6.2×10^{-2}
	180	7	{ 250 300	{ -110. -15.
	90	7	300	9.2

Table 2. Variation of Sway for Unsteered Ship.

Briefly the system was unstable and the ship was turned around. There is no chance to control the ship. Drift was negative 110 ft in 4 minute but this value was reduced the negative 15 ft in 5 minute. The summarization of this test can be stated as the following: computer program (II) responses were tabulated for 3 different wind directions and various velocities. According to the table, wind disturbance was not so important on the course from any direction until 5 miles/hr wind velocity was reached. Critical points were 5 and 7 miles/hr wind velocities for stern and beam wind respectively.

Second test: 15 degree rudder displacement was applied under the beam and stern wind disturbances. The main purpose was to show the effect of the rudder displacement on the ship course with wind. As is known, positive rudder displacement causes negative yaw. All responses are shown in Figures 25-30 and tabulated values are shown in Tables 3 and 4. According to the Tables 3 and 4, when the ship is effected 5 miles beam wind velocity and positive 15 degree rudder displacement. So, ship was deflected -327 degree from the original path in 5 minute. Drift was 80 ft in 2.5 minute and -100 ft in 5 minute. As mentioned earlier, 5 miles/hr beam wind with zero rudder displacement has created 8×10^{-3} rad. deviation from the original path, as shown in Table 1. But the same condition with 15° rudder displacement has -327° deviation from the original path. In this case, the ship was so effected with rudder force and started to turn around the path.

Figure	WD.DI.(degree)	WD.VEL.(miles)	Time(sec)	Yaw(rad)
	180	3	300	-4
	180	5	300	-18
	90	5	300	-5.7

Table 3. Variation of Yaw for Manually Steered Ship (S=15°).

Figure	WD.DI.(degree)	WD.VEL.(miles)	Time(sec)	Sway(ft)
	180	3	{85 240	{9 -18
	180	5	nonlinear oscillation	
	90	5	{150 300	{80 -100

Table 4. Variation of Sway for Manually Steered Ship (S=15°).

The other situation is a stern wind disturbance with 15° rudder displacement. Also known from Table 1, 5 miles/hr stern wind velocity with zero rudder displacement has created -.04 rad. deviation from the original path. In this case, 15 degree rudder displacement created -18 rad. deviation in 5 minute. Ship was turned 360° around the course in 2 minute and drifted with divergent oscillation.

All these tests were made with ship speed being 1 mile/hr in the calm water and values of wind velocities were suddenly applied.

B. AUTOMATICALLY STEERED SHIP

Earlier tests show that the ship was effected by wind and rudder.

In this section, another test was developed to minimize the deviation from the original path. For this reason rudder deflection is used and is written as a function of ψ.

The equation for the rudder deflection is:

$$\bar{\delta} = \alpha \dot{\bar{\delta}}d + \bar{\delta}d = k_t \dot{\psi} + k\psi$$

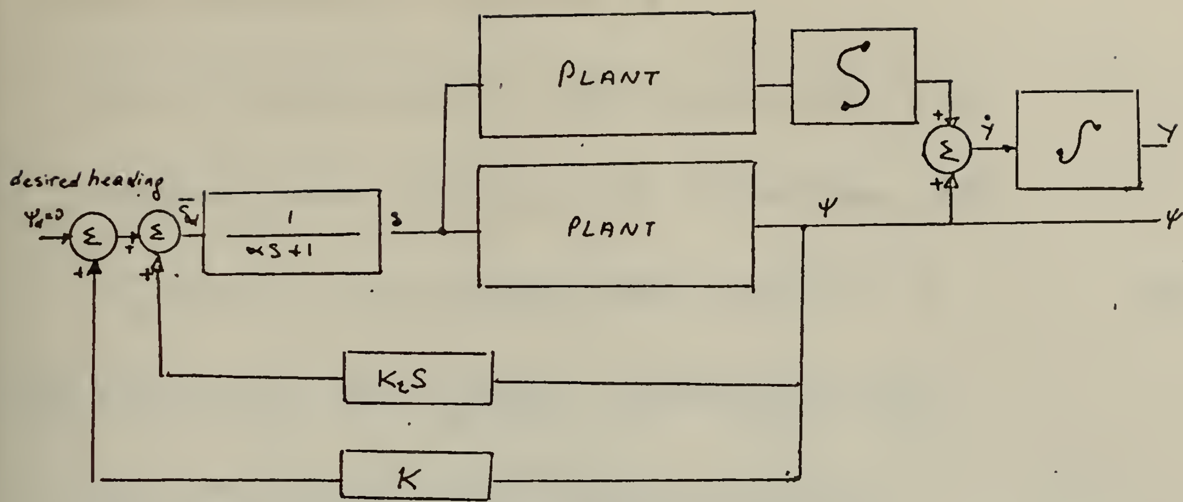
where

α = rudder response time constant (time lag)

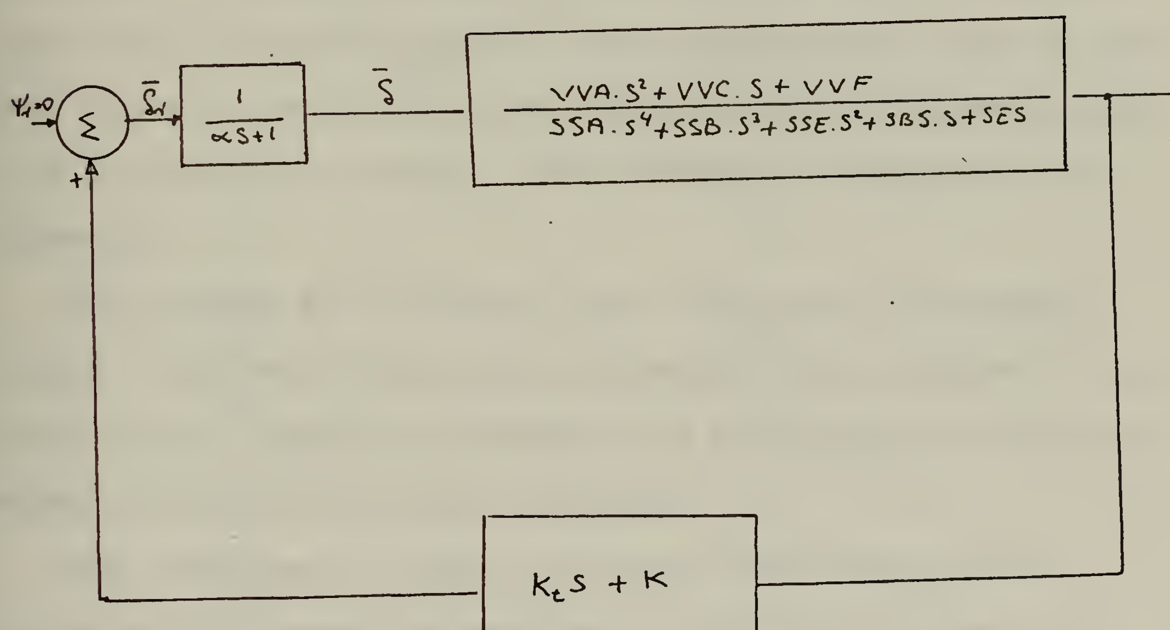
k = yaw gain constant

k_t = yaw rate gain constant

Thus, the system would be:



These control effectors are the function of some parameter that could be measured by sensors, usually a motion parameter.



Let

$$\frac{1}{\alpha} = \alpha_1$$

$$G = \frac{\psi}{\delta d} = \frac{VVA.S^2 + VVC.S+VVF}{SSA.S^4+SSB.S^3+SSE.S^2+SBS.S+SES} \cdot \frac{\alpha_1}{S+\alpha_1} \qquad (26)$$

$$H = K + K_t S$$

So the characteristic equation is

$$SSA.S^5+(SSB+\alpha_1/SSA).S^4+(SSE+\alpha_1 SSB-\alpha_1.VVA.K_t)S^3$$

$$+(SBS+\alpha_1.SSE-\alpha_1.VVC.K_t-\alpha_1.VVA.K)S^2+(SES+\alpha_1.SBS$$

$$-\alpha_1.VVF.K_t-\alpha_1.VVC.K)S+(\alpha_1.SES-\alpha_1.VVF.K) = 0 \qquad (27)$$

For simplicity, let B_1=SSA, B_2=SSB+α_1·SSA etc.

$$B_1\cdot S^5+B_2 S^4+(B_3-B_4\cdot K_t)S^3+(B_5-B_6\cdot K_t-B_7 K)S^2$$

$$+(B_8-B_9\cdot K_t-B_{10}\cdot K)S+(B_{11}-B_{12}\cdot K) = 0 \qquad (28)$$

This characteristic equation was calculated using computer program 1. Computer program III was used to get transient responses in yaw and sway for the automatically steered ship. Each curve of yaw or sway was developed with different parameter values (for K and K_t), for different wind speed and direction.

The problem is to minimize the variation on the ship course. For this reason, the parameter plane program is applied to the system to determine the best value of yaw gain constant and yaw rate gain constant.

The responses of these are shown from Figure 40 to Figure 44 for several situations. The values chosen are the minimum values shown on the curves according to reference [6].

The parameter plane outputs are developed by applying equation (27) to program IV. Table 5 shows the best values

Wind Direction	Wind Velocity	K	Kt
000	1	2.493	3.53
000	3	2.327	3.712
000	5	4.994	3.722
090	3	3.945	3.480
180	1	3.425	3.577

Table 5. Yaw Gain Constant and Yaw Rate Gain Constant Values for Several Wind Directions and Velocities (which were found from parameter plane computer outputs).

of the yaw gain constant and yaw rate gain constant for the automatically steered ship.

All responses of the automatically steered ship are shown in Figure 31 through 33 and tabulated on Tables 6 and 7.

According to the Tables (6 and 7), the variations on the course is minimized for the low wind velocities. In the head wind disturbance, course variations were reduced to 0 rad. in 5 minute.

One mile/hr wind velocity response reached 0 rad. in 20 sec., 3 mile/hr gave 0 rad. response in 65 sec. But 5 miles/hr wind velocity response was 1×10^{-4} rad. in 5 minute.

For the beam wind effection, 3 miles/hr wind velocity was applied to the ship. Deviation was 0 rad. in 70 sec.

A stern wind was also tried, 1 mile/hr wind velocity caused a deviation which was corrected to 0 rad. in 65 sec.

As is seen above, the ship trajectory was kept on the original path. This desired course or original path was assumed to be due North. Also the same test condition was applied.

Figure	WD.DI.(degree)	WD.VEL.(miles)	Time(sec)	Yaw(rad)
	0	1	{ 10 300	{ $-1\mathrm{x}10^{-4}$ ~0
	180	1	{ 50 300	{ $-1\mathrm{x}10^{-4}$ ~0
	0	3	{ 15 300	{ $-1\mathrm{x}10^{-4}$ ~0
	90	3	{ 20 300	{ $.85\mathrm{x}10^{-4}$ ~0
	0	5	{ 30 300	{ $1\mathrm{x}10^{-4}$ $.5\mathrm{x}10^{-4}$

Table 6. Variation of Yaw for Automatically Steered Ship.

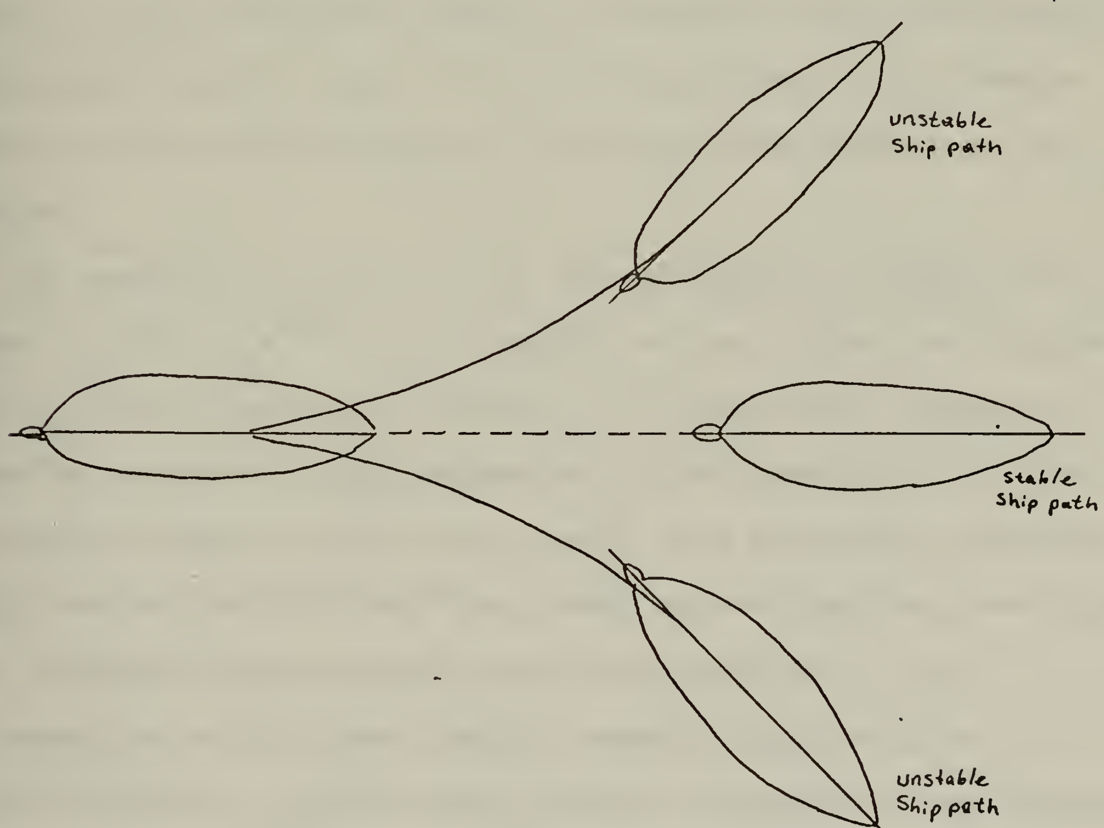
Figure	WD.DI.(degree)	WD.VEL.(miles)	Time(sec)	Sway(ft)
	0	1	300	$10\mathrm{x}10^{-4}$
	180	1	300	$13\mathrm{x}10^{-3}$
	0	3	300	$11.5\mathrm{x}10^{-4}$
	90	3	300	$-5\mathrm{x}10^{-3}$

Table 7. Variation of Sway for Automatically Steered Ship.

IV. STABILITY ANALYSIS

Stability may be defined in the following way. Establish an equilibrium situation, disturb the equilibrium condition with arbitrary disturbances. If the ship returns or tends to return to the original straight-line condition of equilibrium after a disturbances of the smallest amount (infinitesimal disturbance), it is stable. If it departs or has the tendency to depart from the original straight-line equilibrium condition, it is unstable.

The unstable ship will go in to a starboard or port turn.



Stability analysis is performed by solving for the eigen values of the system matrix (4 x 4) which is defined on page 46. Reference [3] contains straight forward algebraic solution of the linearized equations. According to reference [3] a complete algebraic solution of linearized equation (20) is obtained with a differential operator. Solution result of any independent variable X_j is defined as:

$$X_{j(t)} = C_{ij}e^{\sigma_1 t} + C_{2j}e^{\sigma_2 t} + \ldots + C_{mj}e^{\sigma_m t}$$

$$X_{j(t)} = \sum_{i=1}^{m} C_{ij}\, e^{\sigma_i t} \qquad (29)$$

Where C_{ij} are arbitrary constants depending upon the initial conditions and $\sigma_1, \sigma_2, \ldots \sigma_m$ are the roots of the determinant of the coefficients in the linearized equations of motion.

In general $\sigma_1, \sigma_2, \ldots \sigma_m$ are different in value, the terms $C_1e^{\sigma 1t}$, $C_2e^{\sigma 2t}$, ... $C_me^{\sigma mt}$ can not negate one another. The only way that each term can go to zero with increasing time is for each of the exponents to be negative. If σ is a complex number in the form $\sigma=a+ib$, the following relationship hold $e^{\sigma t}=e^{(a+ib)t}=e^{at}(\cos bt+i \sin bt)$ and the condition for stability requires that the real parts of $\sigma_1, \sigma_2, \ldots \sigma_m$ be negative as they are complex numbers (the imaginary σ, parts of number indicate the angular frequency of oscillation). The Routh oriterian can also be applied to determine system stability. If we look at equation (20) again, the automatically steered ship equation would be:

$$ABC.\ddot{\bar{\psi}}-AAD.\dot{\bar{\psi}}-BFA.\bar{\psi}-BGC.\bar{V}-BGD.\bar{U} = AAE.\bar{S}$$

$$-BGE.\dot{\bar{\psi}}-BFD\bar{\psi} +ABD.\dot{\bar{V}}-BGF\ \bar{V}-BGG\ \bar{U} = AAC.\bar{S}$$

$$-CAA.\dot{\bar{\psi}}-BFG\bar{\psi}- BGA.\bar{V}+ABE.\dot{\bar{U}}-CAB.\bar{U} = CAC.\bar{S}$$

$$QOQ.\dot{\bar{\psi}}+QQO.\bar{\psi} = QQQ.\dot{\bar{S}} + \bar{S} \qquad (30)$$

Where

$$QQO = k_t, \qquad QOQ = k, \qquad QQQ = tr = \alpha$$

Equation (30) will be written in the following way.

$$\begin{vmatrix} ABC \\ 0 \\ 0 \\ 0 \end{vmatrix} \ddot{\bar{\psi}}+ \begin{vmatrix} -AAD \\ -BGE \\ -CAA \\ QQO \end{vmatrix} \dot{\bar{\psi}}+ \begin{vmatrix} -BFA \\ -BFD \\ -BFG \\ QOQ \end{vmatrix} \bar{\psi}+ \begin{vmatrix} 0 \\ ABD \\ 0 \\ 0 \end{vmatrix} \dot{\bar{V}}+ \begin{vmatrix} -BGC \\ -BGF \\ -BGA \\ 0 \end{vmatrix} \bar{V}+ \begin{vmatrix} 0 \\ 0 \\ ABE \\ 0 \end{vmatrix} \dot{\bar{U}}+ \begin{vmatrix} -BGD \\ -BGG \\ -CAB \\ 0 \end{vmatrix} \bar{U}+ \begin{vmatrix} 0 \\ 0 \\ 0 \\ -QQQ \end{vmatrix} \dot{\bar{\xi}}+ \begin{vmatrix} -AAE \\ -AAC \\ -CAC \\ -1 \end{vmatrix} \bar{\xi}=0 \qquad (31)$$

As is seen from equation (31) there are four independent variables namely yaw, sway, surge and the rudder angle. The form used in equation (29) is applied.

$$\psi=\sum_{i=1}^{m} \bar{\psi}_i e^{\sigma_i t},\quad \bar{V}=\sum_{i=1}^{m} \bar{V}_i e^{\sigma_i t},\quad \bar{U}=\sum_{i=1}^{m} \bar{U}_i e^{\sigma_i t},\quad \bar{S}=\sum_{i=1}^{m} \Delta_i e^{\sigma_i t} \qquad (32)$$

Where $\bar{\psi}_i$, $\bar{V}_i$, $\bar{U}_i$ and Δ_i are arbitrary constants.

Substitution of equation (32) into equation (31):

$$\begin{vmatrix} ABC\sigma_i^2-AAD.\sigma_i-BFA \\ -BGE\sigma_i-BFO \\ -CAA\sigma_i-BFG \\ QQO\sigma_i+QOQ \end{vmatrix} \bar{\psi}_i+ \begin{vmatrix} -BGC \\ ABD\sigma_i-BGF \\ -BGA \\ 0 \end{vmatrix} \bar{V}_i+ \begin{vmatrix} -BGD \\ -BGG \\ ABE\sigma_i-CAB \\ 0 \end{vmatrix} \bar{U}_i+ \begin{vmatrix} -AAE \\ -AAC \\ -CAC \\ -QQQ\sigma_i-1 \end{vmatrix} \bar{\Delta}_i= 0 \qquad (33)$$

Solutions of equation (33) are possible for only values of σ_i which satisfy equation (34).

$$\begin{vmatrix} (ABC\sigma_i^2-AAD\sigma_i-BFA) & (-BGC) & (-BGD) & (-AAE) \\ (-BGE\sigma_i-BFD) & (ABD\sigma_i-BGF) & (-BGG) & (-AAC) \\ (-CAA\sigma_i-BFG) & (-BGA) & (ABE\sigma_i-CAB) & (-CAC) \\ (QQO\sigma_i+QOQ) & 0 & 0 & (-QQQ\sigma_i-1) \end{vmatrix} = 0 \qquad (34)$$

Program I gives the solution of this equation as fifth order polynomial. For different situations such as wind velocity and direction other polynomials were also found. As is pointed out in page , it is important for the real root to be negative for system stability.

This test was examined in two situations:

Situation 1: constant wind directions with varied velocities.

Situation 2: constant wind velocities with varied direction.

The first situation was divided into 3 sections:

Section a: constant head wind with several velocities.

Section b: constant beam wind with several velocities.

Section c: constant stern wind with several velocities.

The second situation was divided into 2 section:

Section a: constant 7 miles/hr wind velocity with several directions.

Section b: constant 9 miles/hr wind velocity with several directions.

In order to examine the stability of an automatically steered ship in wind, all these situations are solved with computer

program I, tabulated on Tables 8 - 12 and drawn on the CALCOMP plotters, from Figures 45 through 49.

Situation 1: This test was developed, constant wind direction disturbed the ship. Wind velocities changed each time.

a) Head wind disturbance:

As is shown in Table 8 and Figure 45, the ship would be stable in varied head wind velocities until 13 miles/hr wind velocity. After this range, the ship could be unstable on the horizontal plane.

b) Beam wind disturbance:

In this case, the stability range was shorter (from 1 to 8 miles/hr). After the 8 miles/hr wind velocities the ship could be strictly unstable, shown on Table 9 and Figure 46.

c) Stern wind disturbance:

Stern wind effect on the stability might be the shortest range, as shown on Figure 47 and Table 10. This range included from 1 to 6 miles/hr. After 6 miles/hr wind velocities, the ship could be unstable.

Situation 2: In this case, constant wind velocity disturbed the ship. Wind direction was changed for each test.

a) In 7 miles/hr wind velocity with varied direction. From 0 degree through 180 degree wind directions were tried in 7 miles/hr wind velocity. According to Table 11 and Figure 48, the ship could be unstable when the wind disturbed the ship from 30 degree, 105 degree and 180 degree

wind direction. In the other wind direction and 7 miles/hr wind velocity the ship trajectory could be stable.

b) In 9 miles/hr wind velocity with varied wind direction. As is shown on Figure 49 and Table 12, 9 miles/hr wind velocity with each direction were applied to the system. The ship could be stable when the wind direction was head, 60 degree - 75 degree and 120 degree - 135 degree. For the rest of the directions (0° - 180°) the ship trajectory was unstable.

All of these results show that some of the wind directions and velocities played a very active role. These directions are 30 degree, 105 degree and 180 degree. After the 9 miles/hr wind velocities, many of the wind directions had adverse affects on stability.

Head Wind

Ua	Re σ_1	Re σ_2	Re σ_3	Re σ_4	Re σ_4
5	-8.8	-.111	-6.35×10^{-3}	-12.11	-59.97
6	-8.17	-.117	-9.87×10^{-3}	-11.33	-60.37
7	-7.21	-.124	-1.39×10^{-2}	-11.11	-60.25
8	-6.2	-.13	-1.88×10^{-2}	-10.8	-60.34
9	-5.1	-.138	-2.4×10^{-2}	-10.7	-60.28
10	-3.93	-.148	-2.98×10^{-2}	-10.48	-60.41
11	-2.67	-.162	-3.6×10^{-2}	-10.61	-60.02
12	-1.49	-.189	-4.2×10^{-2}	-10.42	-60.35
13	-4.25×10^{-2}	-.2	-.2	-10.46	-60.09
14	+.964	$-.84 \times 10^{-1}$	$-.84 \times 10^{-1}$	-10.4	-60.4

Table 8. Real Roots of Characteristic Equation.

Beam Wind

Ua	Re σ_1	Re σ_2	Re σ_3	Re σ_4	Re σ_5
1	-9.72	-0.091	-6.4×10^{-4}	-16.3	-59.9
2	-9.65	-0.034	-7.5×10^{-4}	-14.6	-60.66
3	-8.97	-0.099	-9.1×10^{-4}	-13.37	-60.06
4	-8.8	-0.105	-1.46×10^{-3}	-11.1	-55.99
5	-6.76	-0.115	-2.76×10^{-3}	-10.9	-60.25
6	-4.7	-0.129	-4.94×10^{-3}	-10.47	-60.7
7	-2.56	-0.148	-7.8×10^{-3}	-10.4	-60.4
8	-4.6×10^{-3}	-0.239	-0.239	-10.3	-60.2
9	+1.94	-0.025	-0.148	-10.25	-60.4
10	+4.25	-0.029	-0.186	-10.22	-60.42

Table 9. Real Roots of Characteristic Equation.

Stern Wind

Ua	Re σ_1	Re σ_2	Re σ_3	Re σ_4	Re σ_5
1	-9.3	-.09	-4.5×10^{-4}	-13.3	-60.3
2	-8.7	-.0978	-2.43×10^{-4}	-11.7	-60.48
3	-7.3	-.0894	-1.84×10^{-3}	-11.2	-59.95
4	-5.56	-.087	-5.2×10^{-3}	-10.7	-60.14
5	-3.6	-.084	-1.1×10^{-2}	-10.5	-60.3
6	-1.57	-.085	-1.85×10^{-2}	-10.36	-60.47
7	+.443	-.0296	-.0296	-10.286	-60.62
8	+2.57	-.0468	-.0468	-10.24	-60.53
9	+4.67	-.054	-.054	-10.21	-60.55
10	+6.77	-.064	-.064	-10.19	-60.56

Table 10. Real Roots of Characteristic Equation.

WD.DI.	Re σ_1	Re σ_2	Re σ_3	Re σ_4	Re σ_5
0°	-7.21	-.124	-.014	-11.11	-60.25
30°	+3.46	-.0104	-.138	-10.215	-60.44
45°	-3.34	-.082	-.0114	-16.42	-60.38
60°	-4.9	-.088	-.035	-10.63	-60.3
75°	-4.92	-.145	-.0143	-10.6	-60.26
90°	-2.56	-.148	-7.8×10^{-3}	-10.4	-60.4
105°	+3.16	-.0419	-.419	-10.2	-60.56
120°	-6.22	-.0896	-.0153	-10.8	-60.29
135°	-6.56	-.118	-.0249	-10.92	-60.28
150°	-.0113	-.947	-.198	-10.331	-60.364
165°	-1.6×10^{-3}	-.956	-.155	-10.3	-60.44
180°	+.443	-.0286	-.0286	-10.286	-60.62

Table 11. Real Roots of Characteristic Equation, Wind Velocity = 7 miles.

WD.DI.	Re σ_1	Re σ_2	Re σ_3	Re σ_4	Re σ_5
0°	-5.08	-.138	-.024	-10.68	-60.28
30°	+8.53	-.19	-.0238	-10.16	-60.1
45°	+.1054	-1.25	-.0275	-7.85	-60.8
60°	-2.25	-.073	-.0732	-10.4	-60.36
75°	-2.14	-.193	-.0224	-10.42	-60.33
90°	+1.94	-.027	-.145	-10.25	-60.42
105°	+8.14	-.056	-.056	-10.16	-60.62
120°	-3.07	-.07	-.045	-10.45	-60.42
135°	-4.31	-.132	-.038	-10.55	-60.34
150°	+2.8	-.0368	-.17	-10.24	-60.4
165°	+4.1	-.0128	-.146	-10.2	-60.47
180°	+4.67	-.054	-.054	-10.21	-60.55

Table 12. Real Roots of Characteristic Equation, Wind Velocity = 9 miles.

V. DISCUSSION OF RESULTS AND CONCLUSIONS

The procedures for system course control and stability analysis on the horizontal plane were described. Applying the datas which are taken from some references, to the three degree freedom equation gave us course control and stability analysis computer outputs. As a result of these outputs some comments can be made. First of all the unsteered case was examined. It is seen that the stern wind effects the straight course of the ship the most. When the wind velocity reaches 7 miles/hr this effect becomes extremely high. In 5 minutes, the deviation of the course is about 327 degree. At the same time the effect of the head wind on the straight course compared to the stern and beam wind seems to be much less; which is seen from the computer outputs.

To see the effect of the rudder deflection on the straight course there should be positive 15 degree applied to the rudder for beam and stern wind. Under these conditions negative yaw and sway were found. When the ship is under stern wind, applying positive 15 degree rudder deflection will create extremely negative yaw and sway. As it is seen in Figures 29 and 30, when the stern wind velocity reaches 5 miles in 50 seconds, 15 degree rudder deflection will create minus 145 degree deviation from the course.

Computer output results were also obtained for an automatically steered ship. Parameter plane computer programs were applied to the closed loop system for choosing the best value of yaw rate gain constant and yaw gain constant. According to the automatically steered ship computer outputs, in low wind speeds the course variation is minimized. Sway can't be controlled very well but the value of the drift has decreased compared to the unsteered case.

In a final study, using the eigen value theorem, system stability and the critical wind velocity are found and tabulated. Computed results show that 30 degree, 105 degree and 180 degree wind direction are important on the ship course in $\bar{X}$ miles/hr wind velocities. Also 9 miles/hr wind velocity in many direction were so effected.

All the studies done above are made for a ship which has a speed of 1 mile/hr and limited special full loaded ship form.

APPENDIX

Hydrodynamic and aerodynamic coefficients where used in equation (5)

$A_1 = \frac{e}{2} \ell^5 N_{\dot r}'$ $B_1 = \frac{e}{2} \ell^3 Y_{\dot v}'$ $C_1 = \frac{e}{2} \ell^3 X_{\dot v}'$

$A_2 = \frac{e}{2} \ell^3 N_v'$ $B_2 = \frac{e}{2} \ell^2 Y_v'$ $C_2 = \frac{e}{2} \ell^2 X_o'$

$A_3 = \frac{e}{2} \ell^4 N_r'$ $B_3 = \frac{e}{2} \ell^3 Y_r'$ $C_3 = \frac{e}{2} \ell^3 X_{vv}'$

$A_4 = \frac{e}{2} \ell^3 N_s'$ $B_4 = \frac{e}{2} \ell^2 Y_s'$ $C_4 = \frac{e}{2} \ell^2 X_{ss}'$

Where

A_i = Coefficients of hydrodynamic moment in yaw

B_i = Coefficients of hydrodynamic forces in sway

C_i = Coefficients of hydrodynamic forces in surge

$i = 1,2,3,4$

N_1, N_2, Y_1, X_1 are coefficients of aerodynamic forces and moments

Aerodynamic forces and moment

$$X_A' = \frac{X_a}{\frac{ea}{2} \ell^2 Ua^2} = X_i' \, U_a' = X_{a_1}' \cos \psi a$$

$$N_A' = \frac{N_a}{\frac{ea}{2} \ell^3 Ua^2} = N'u'v'+N'v' = \frac{Na_1'}{2} \sin 2\psi a + Na_2' \sin \psi a$$

$$Y_A' = \frac{Y_a}{\frac{ea}{2} \ell^2 Ua^2} = Y_1' \, V_a' = Y_{a_1}' \sin \psi a$$

Which

$$X_1' = X_{a_1}' = -0.015$$

$$Y_1' = Y_{a_1}' = -0.056$$

$$N_1' = N_{a_1}' = -0.0046, \quad N_2' = -N_{a_2}' = -0.017$$

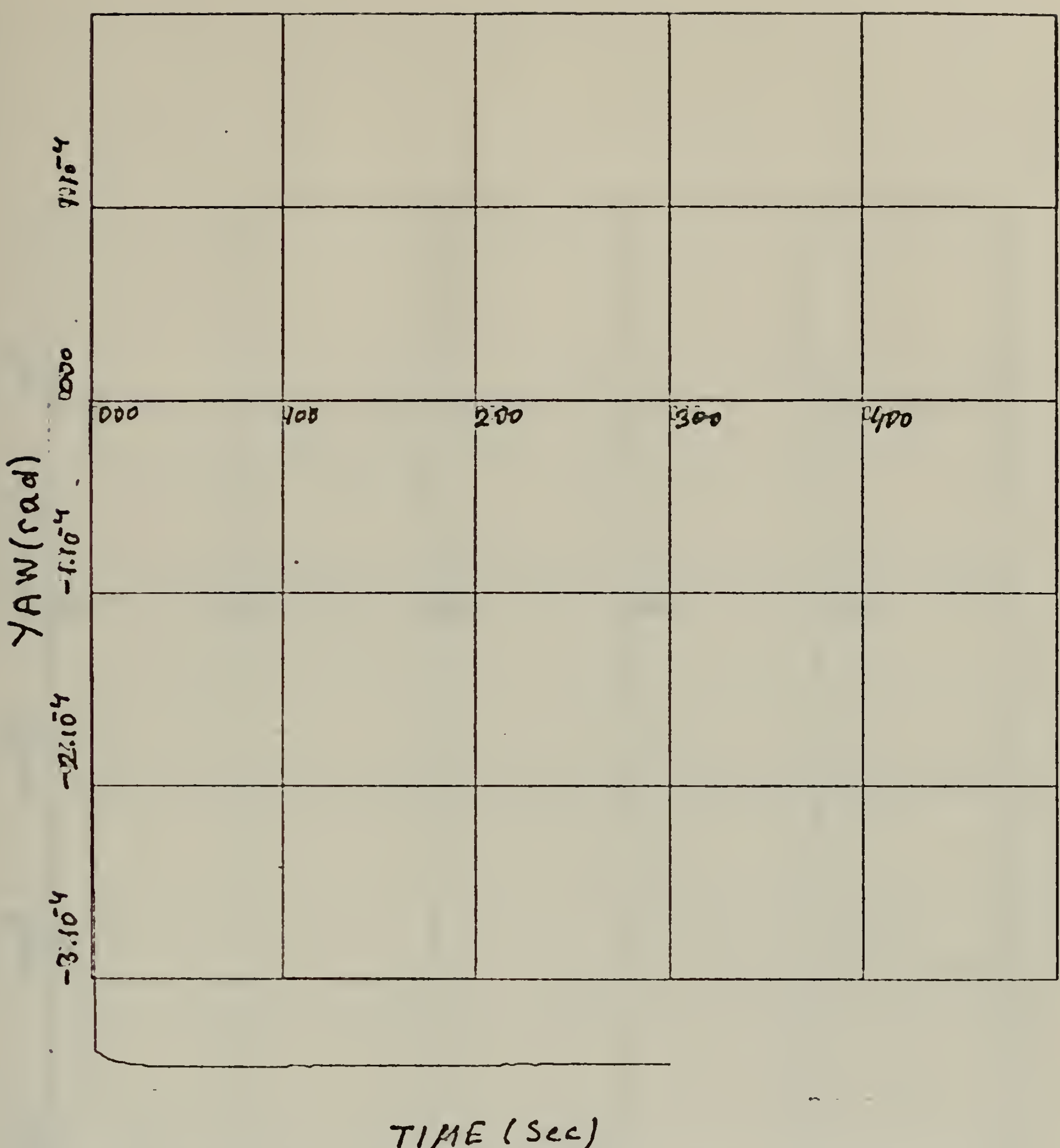


Figure 5. Variation of Yaw Versus Time for Wind Direction 000°, Velocity 1 mile/hr.

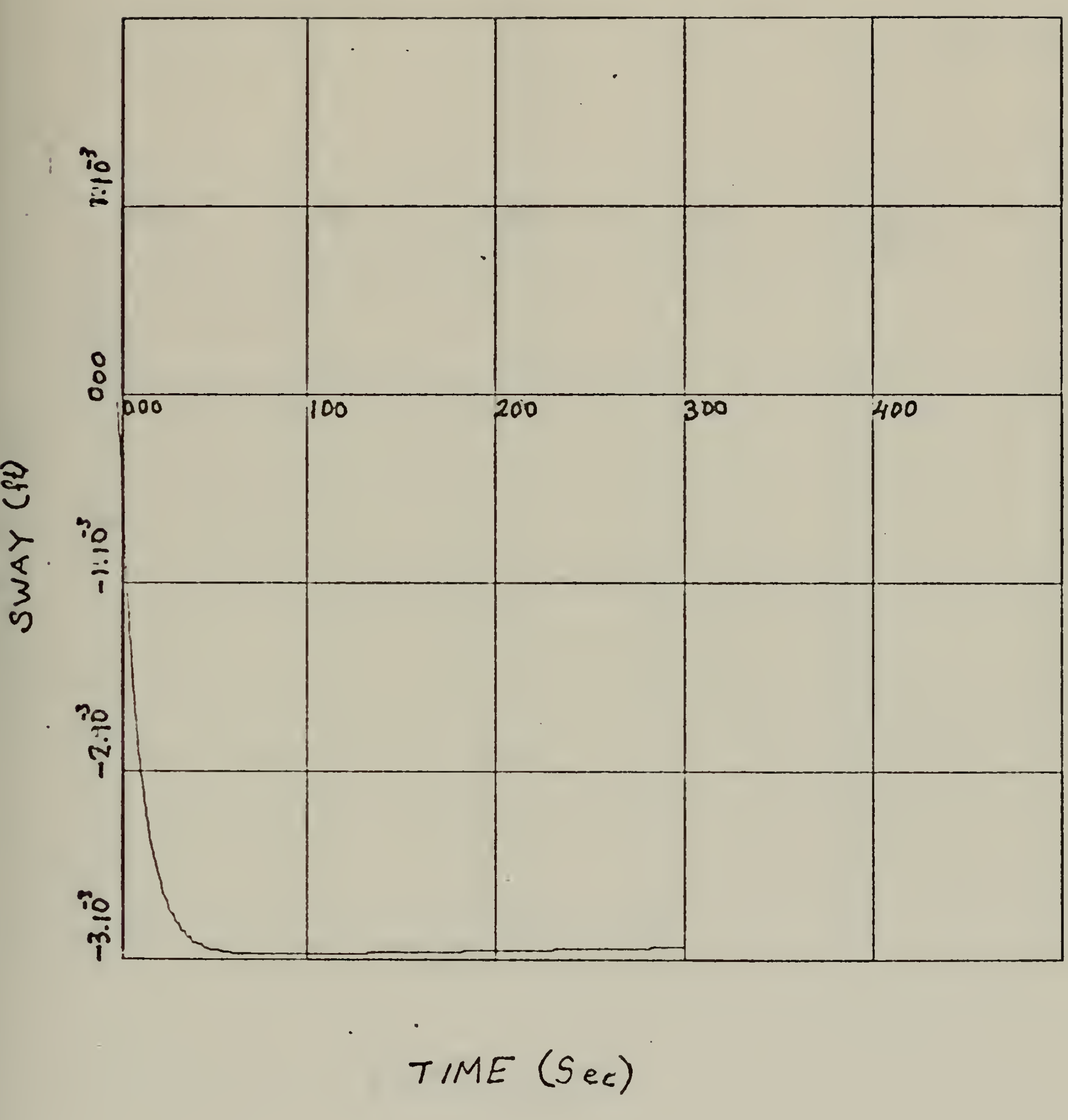


Figure 6. Variation of Sway Versus Time for Wind Direction 000°, Velocity 1 mile/hr.

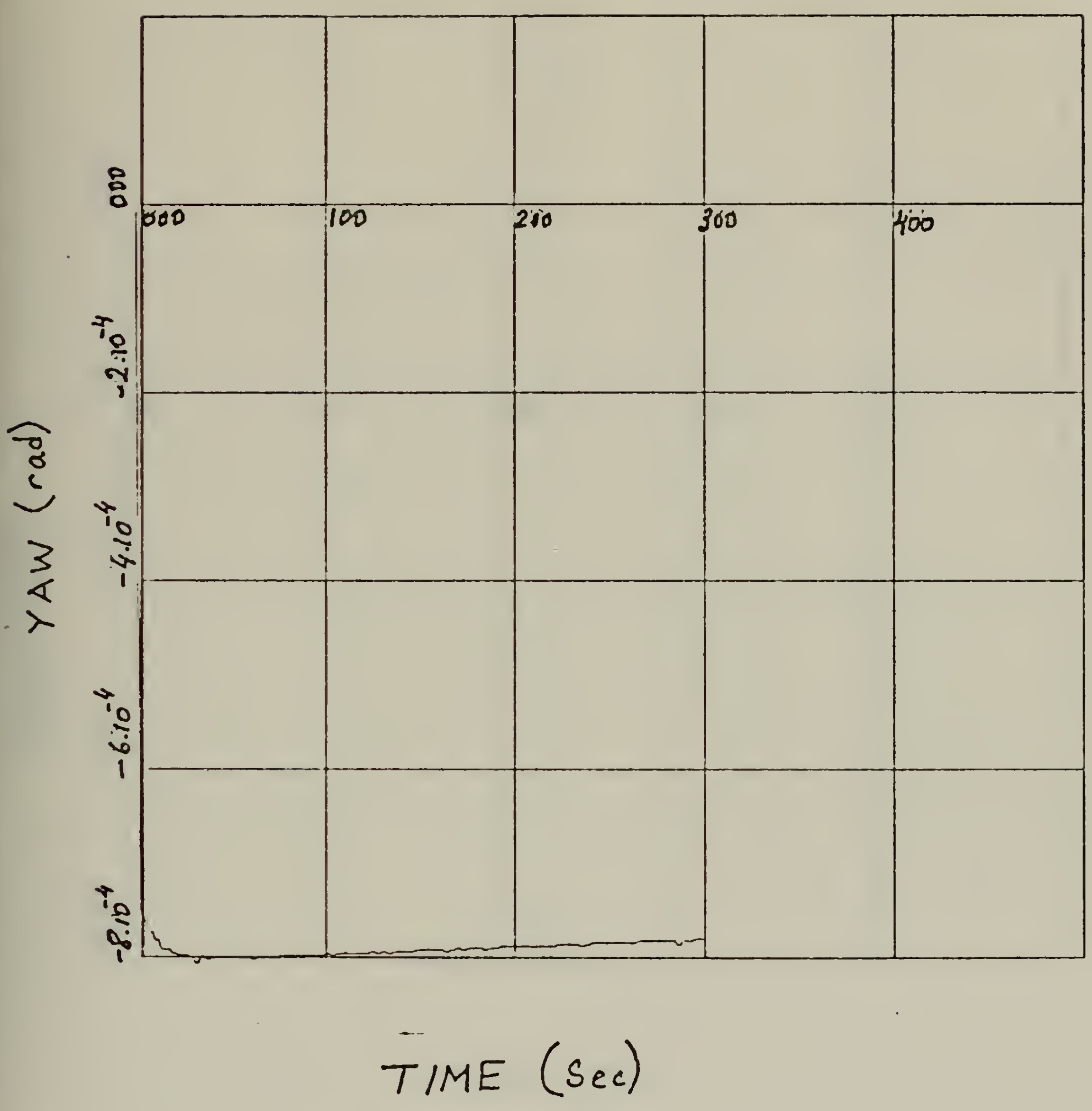


Figure 7. Variation of Yaw Versus Time for Wind Direction 000°, Velocity 3 miles/hr.

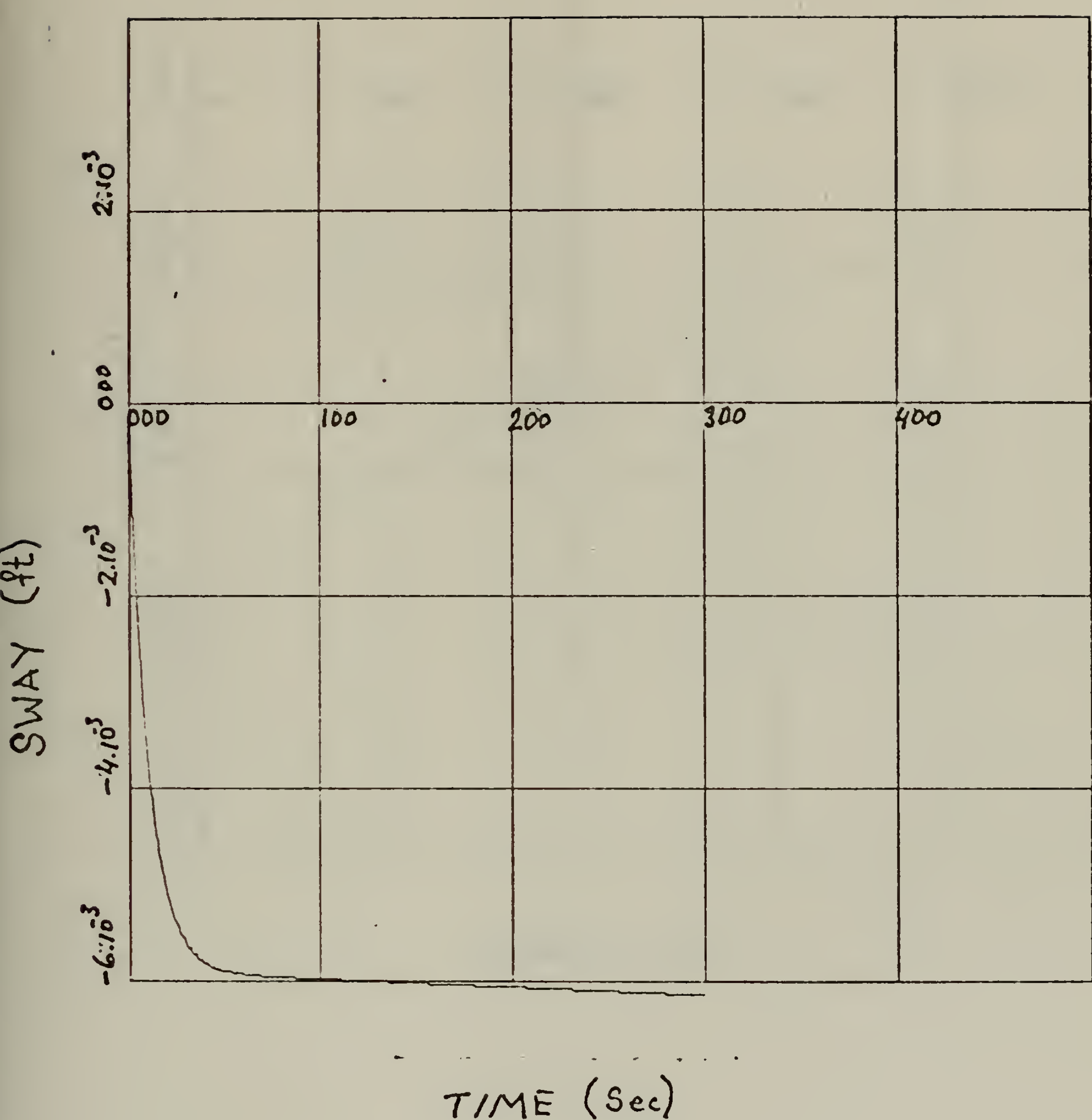


Figure 8. Variation of Sway Versus Time for Wind Direction 000°, Velocity 3 miles/hr.

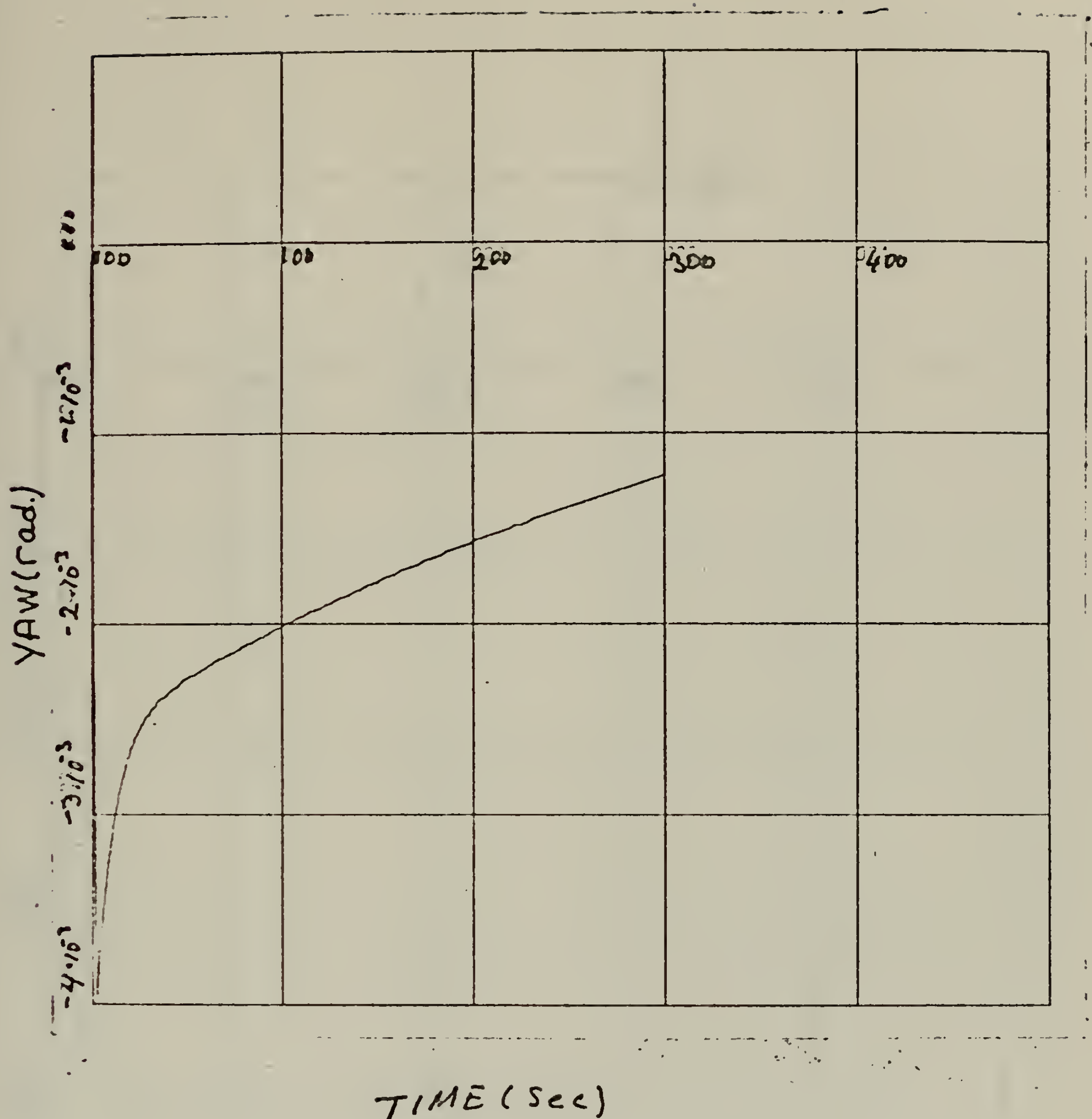


Figure 9. Variation of Yaw Versus Time for Wind Direction 000°, Velocity 5 miles/hr.

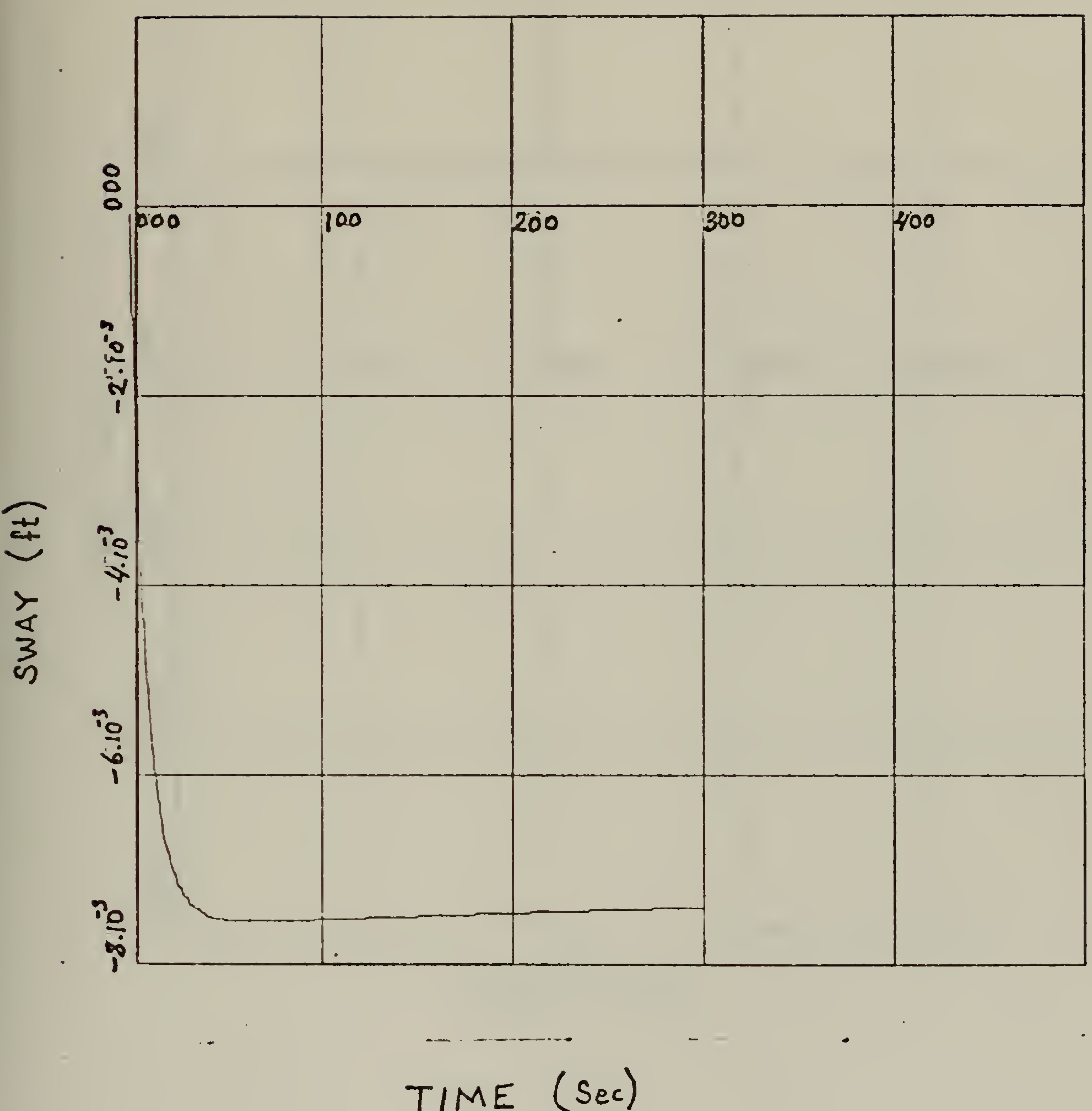


Figure 10. Variation of Sway Versus Time for Wind Direction 000°, Velocity 5 miles/hr.

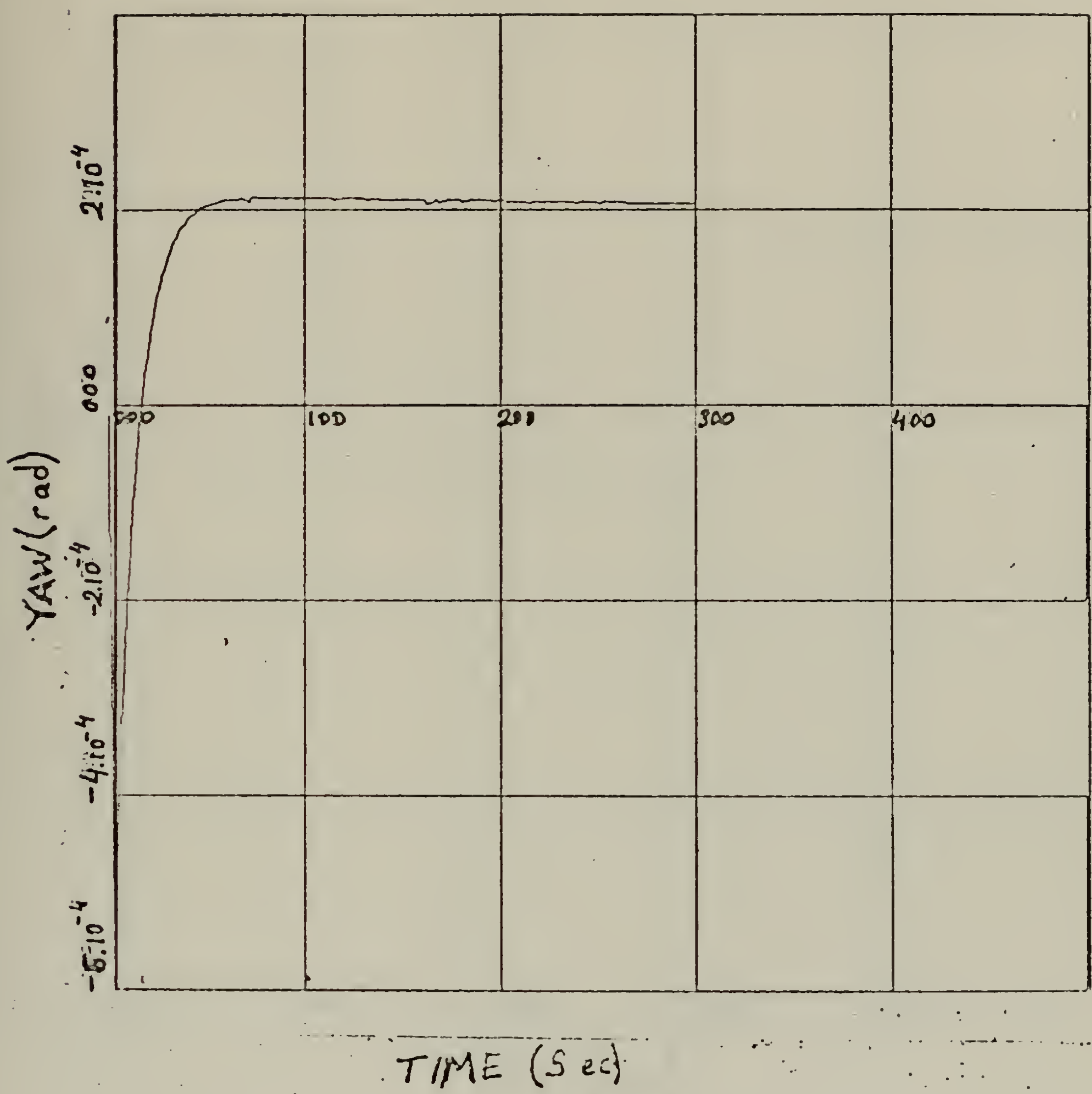


Figure 11. Variation of Yaw Versus Time for Wind Direction 090°, Velocity 3 miles/hr.

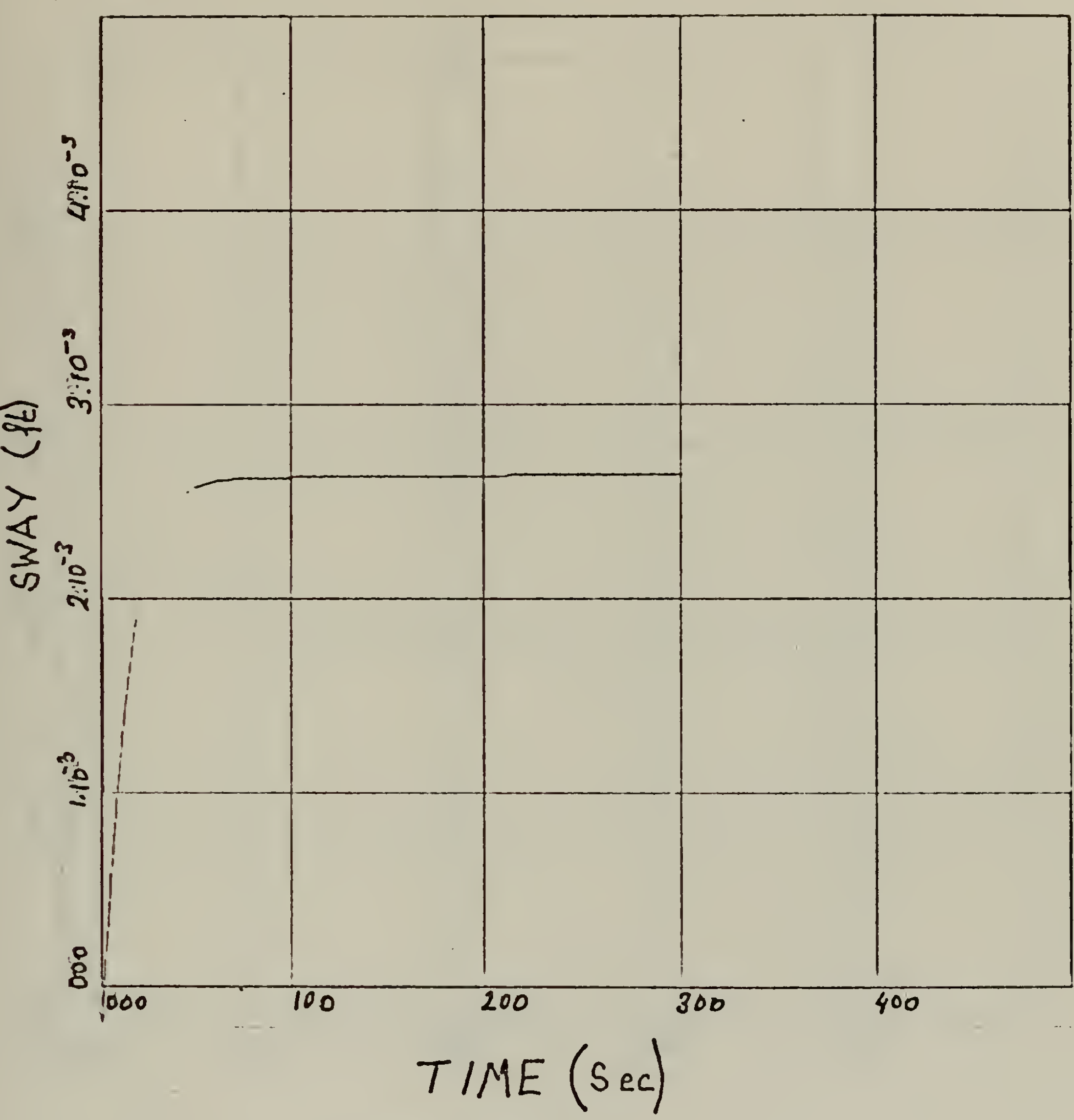


Figure 12. Variation of Sway Versus Time for Wind Direction 090°, Velocity 3 miles/hr.

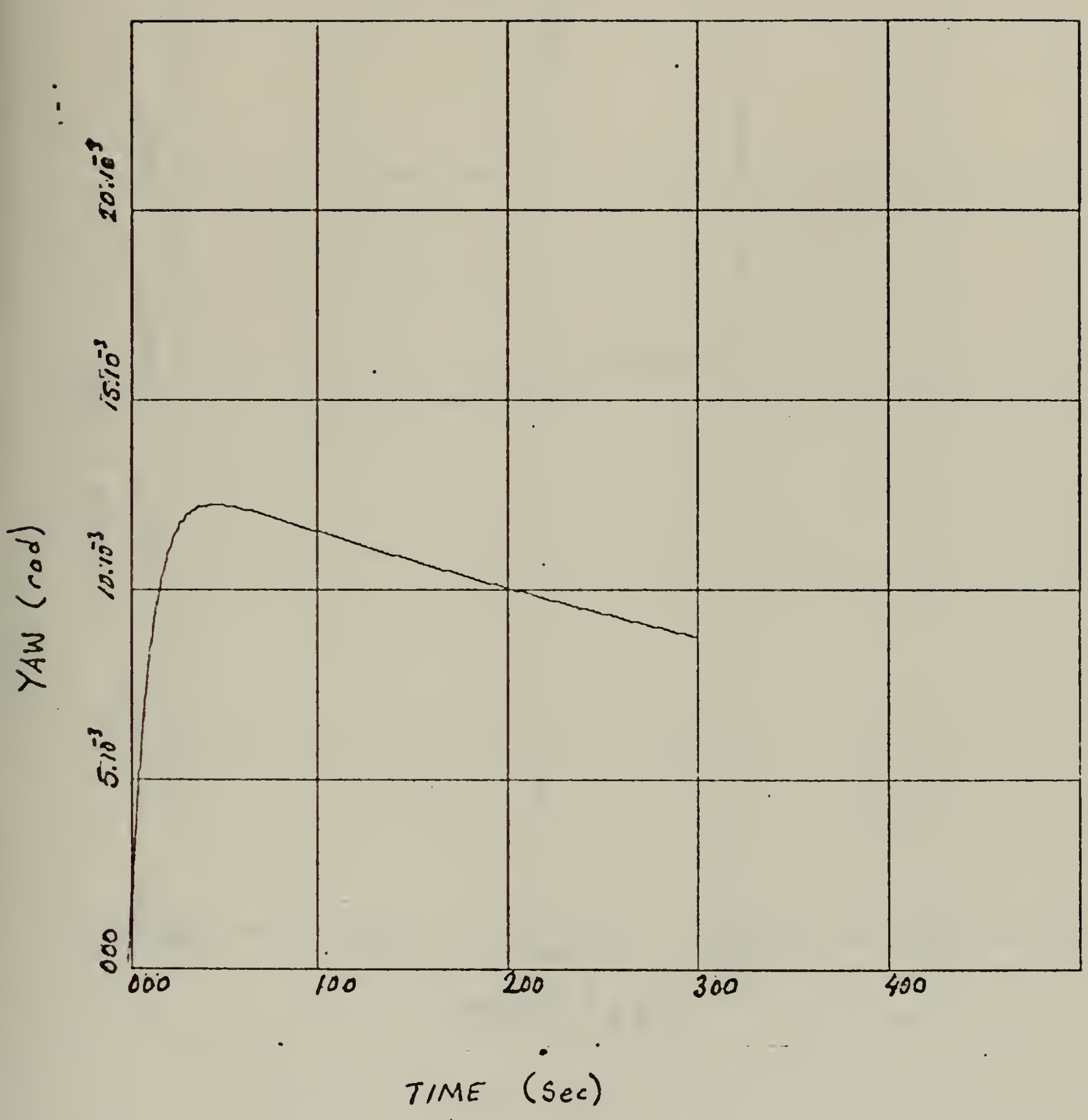


Figure 13. Variation of Yaw Versus Time for Wind Direction 090°, Velocity 5 miles/hr.

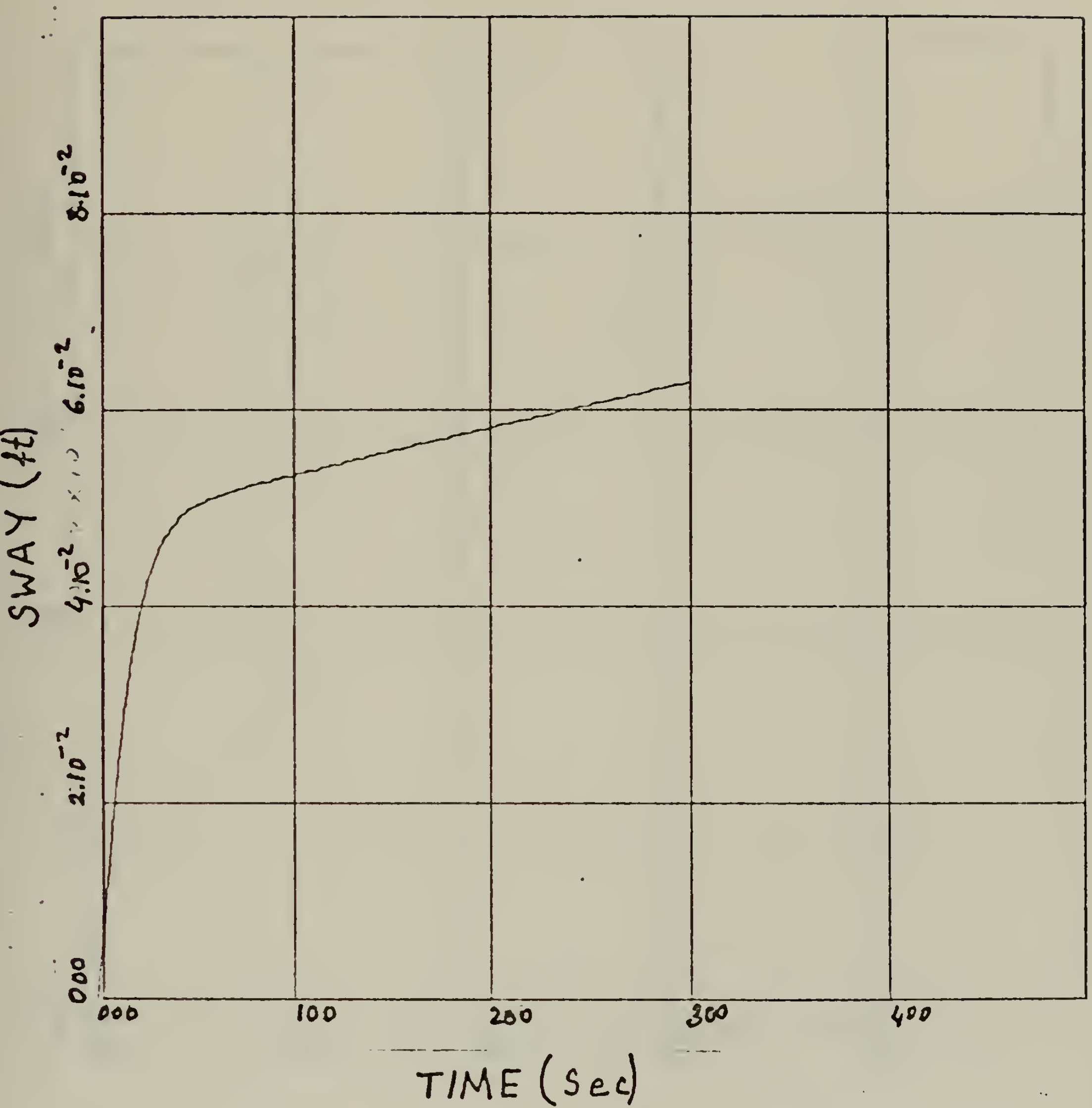


Figure 14. Variation of Sway Versus Time for Wind Direction 090°, Velocity 5 miles/hr.

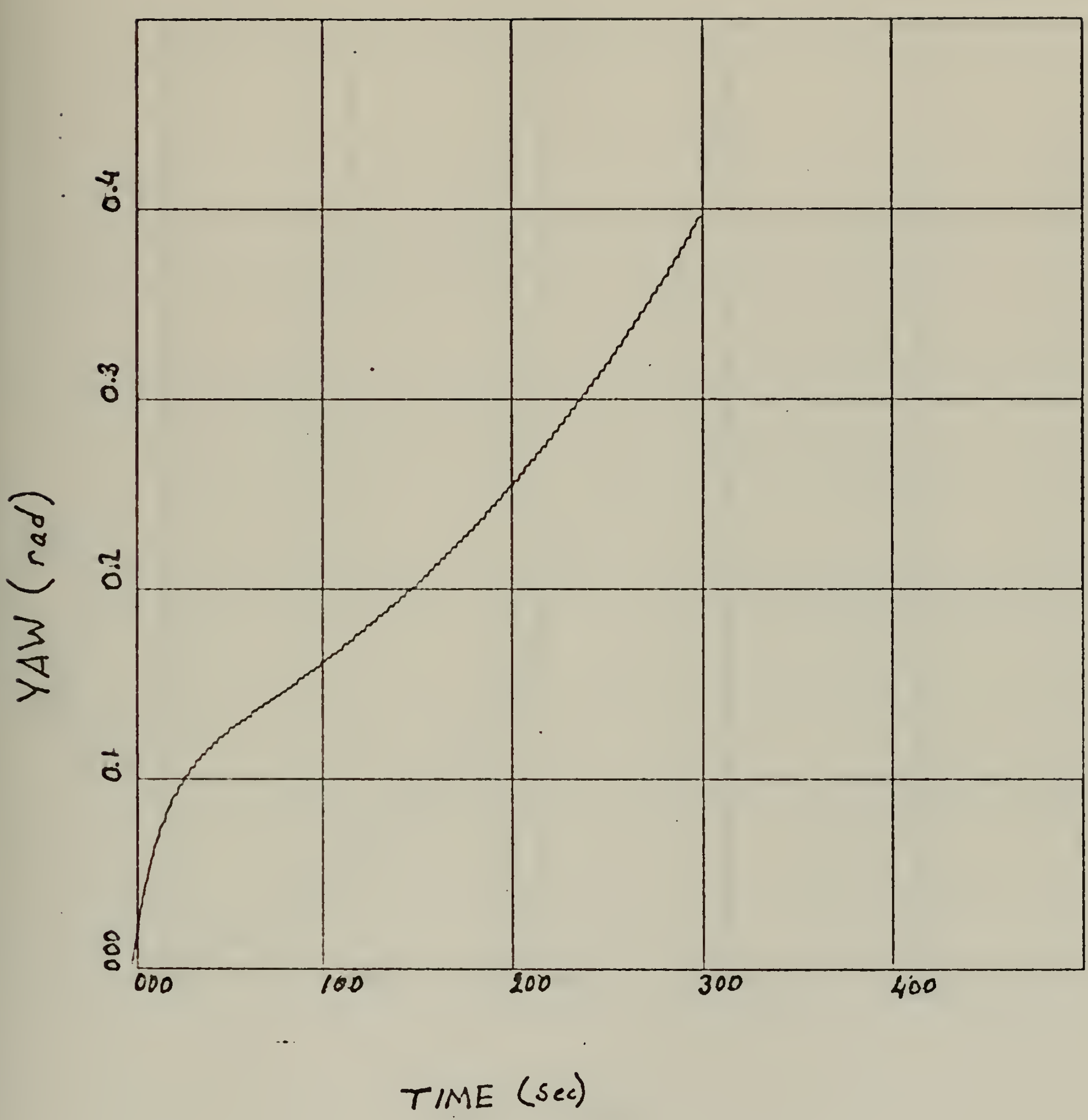


Figure 15. Variation of Yaw Versus Time for Wind Direction 090°, Velocity 7 miles/hr.

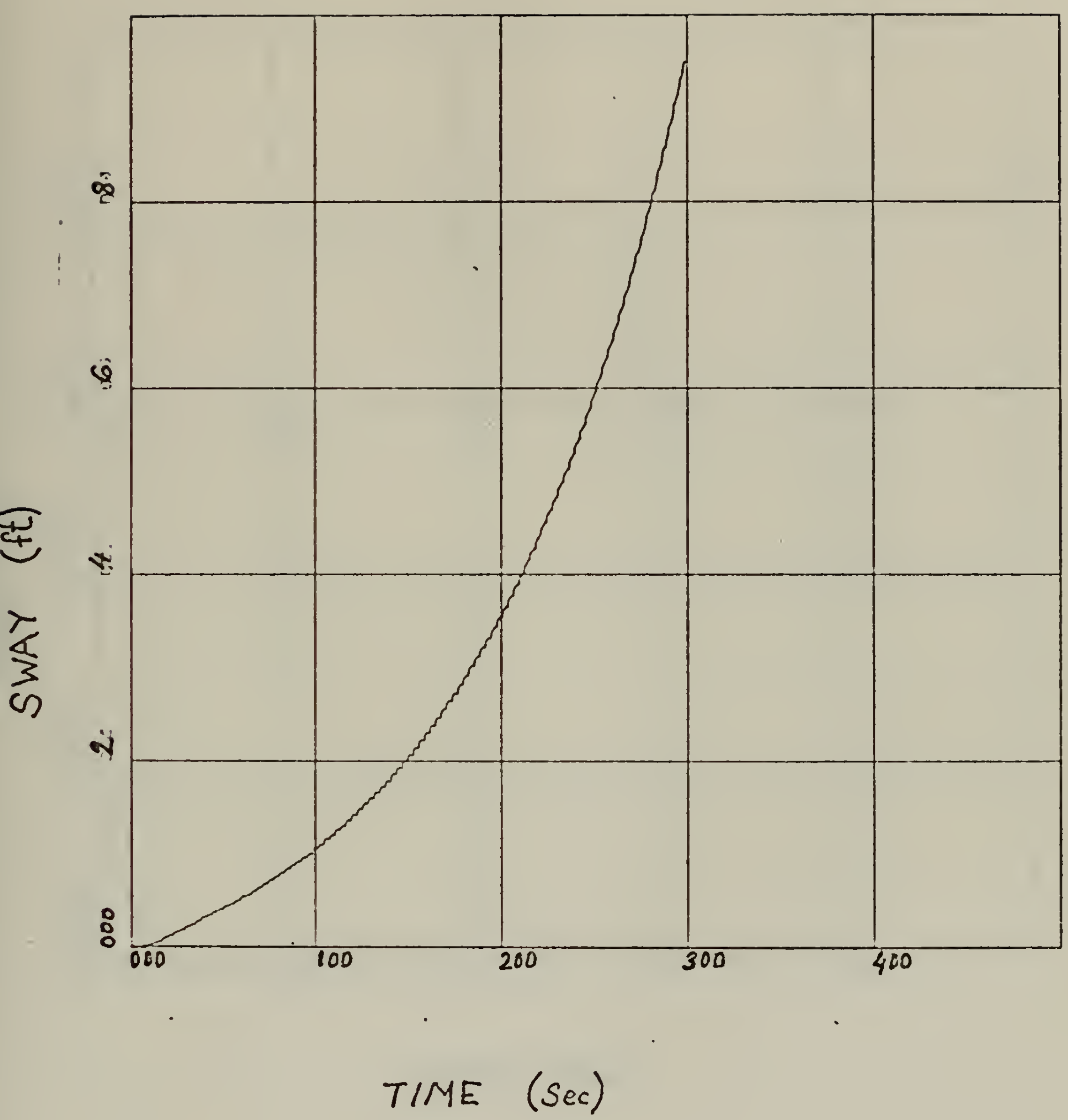


Figure 16. Variation of Sway Versus Time for Wind Direction 090°, Velocity 7 miles/hr.

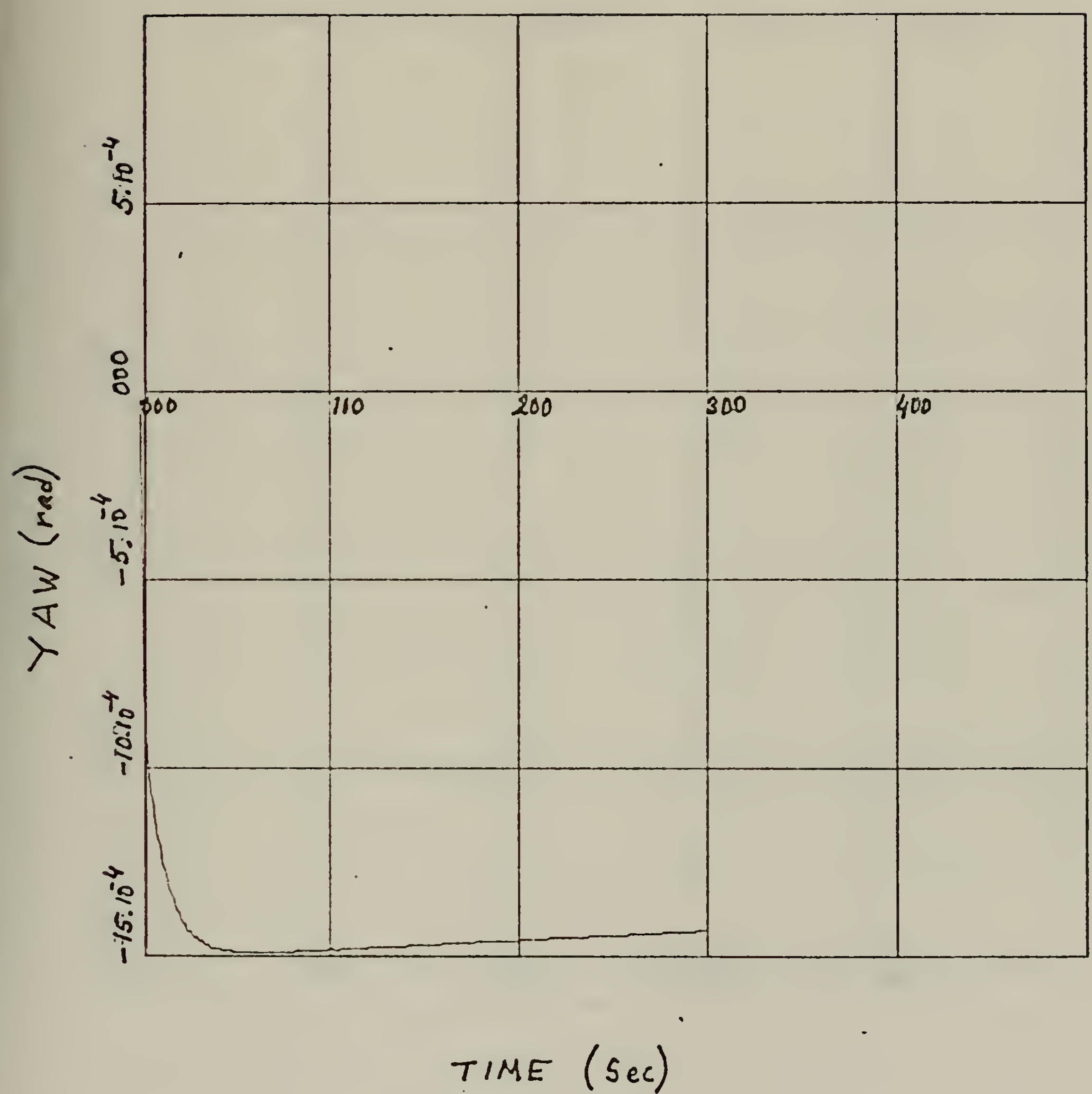


Figure 17. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 1 mile/hr.

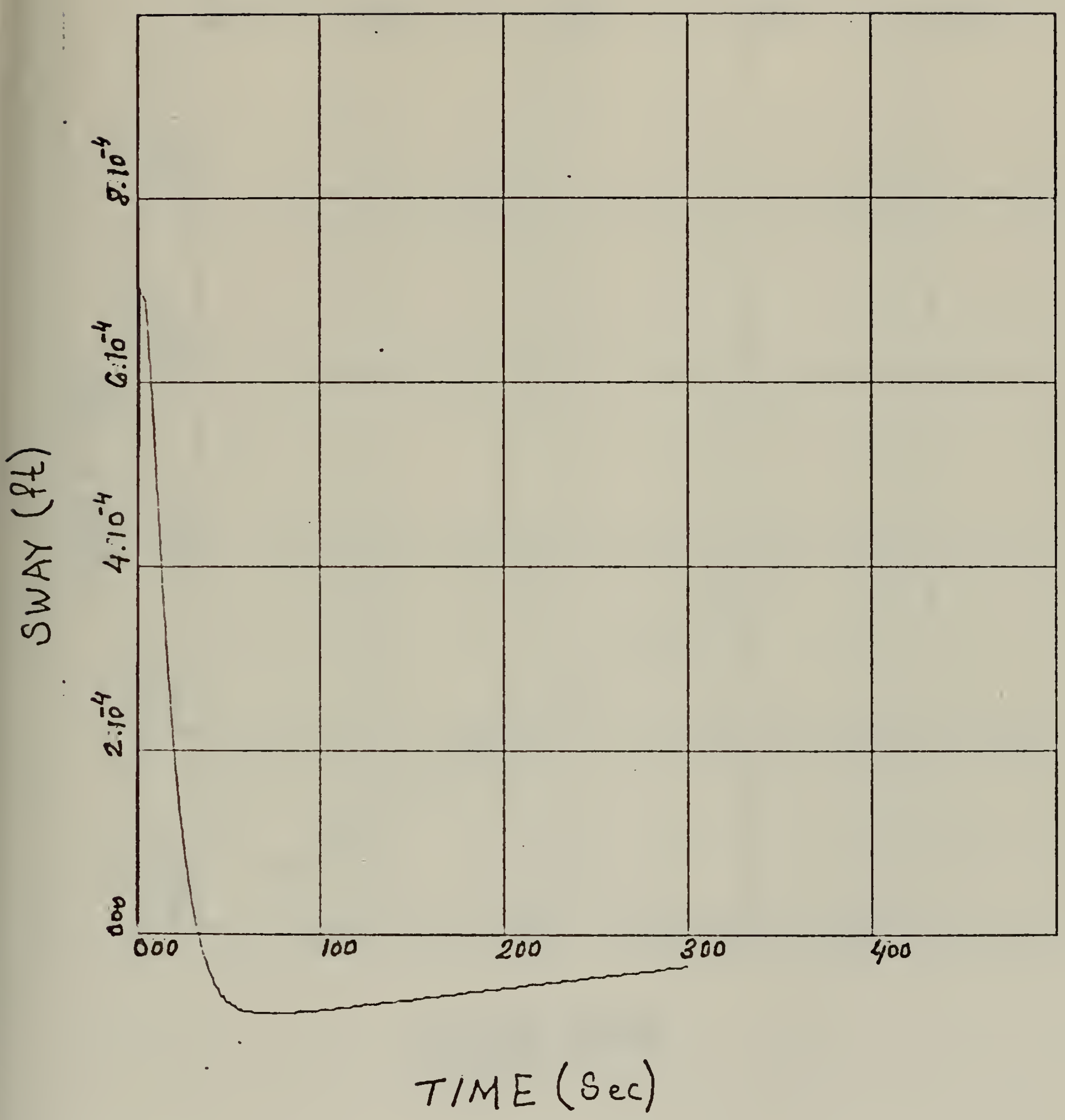


Figure 18. Variation of Sway Versus Time for Wind Direction 180°, Velocity 1 mile/hr.

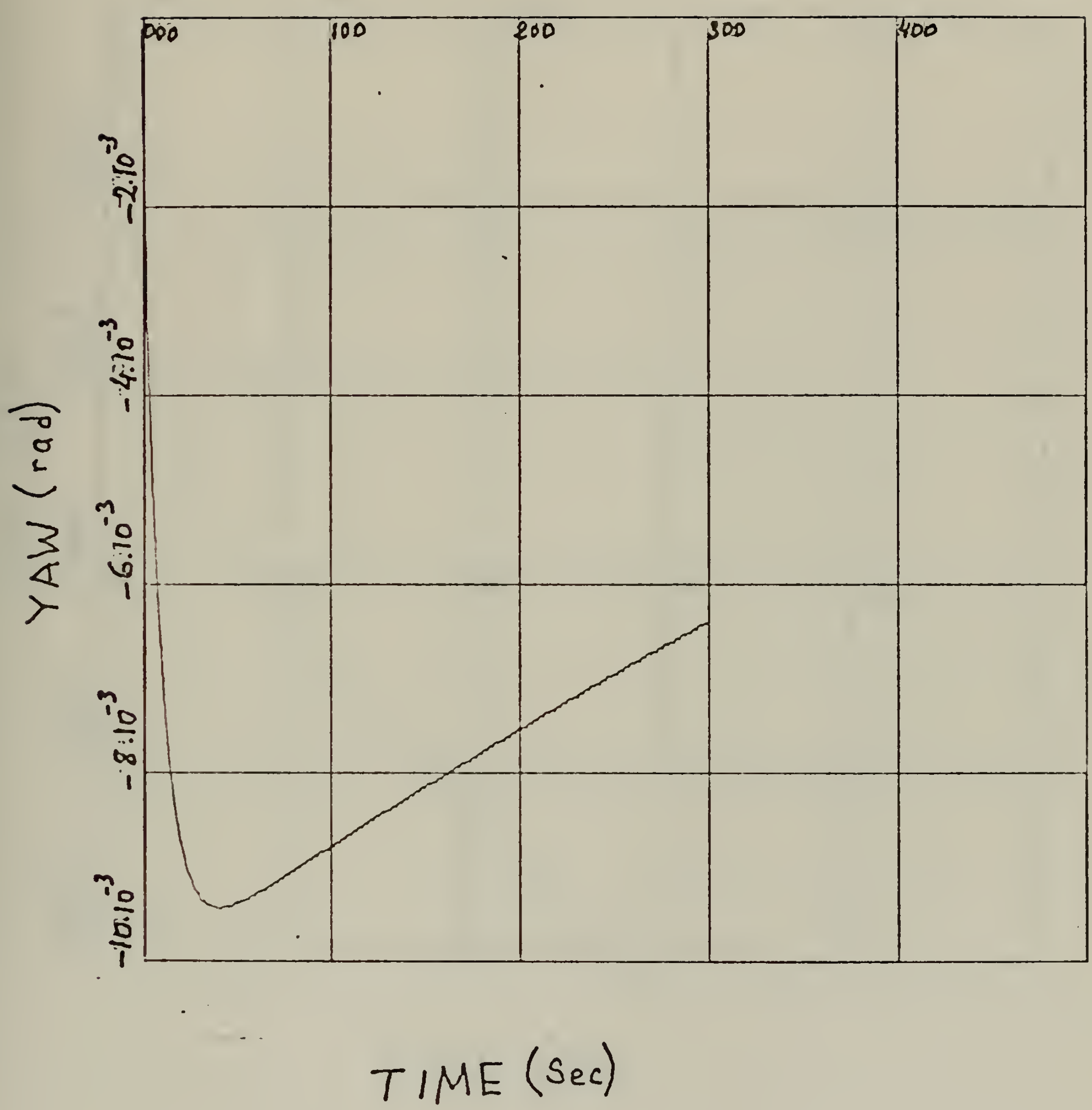


Figure 19. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 3 miles/hr.

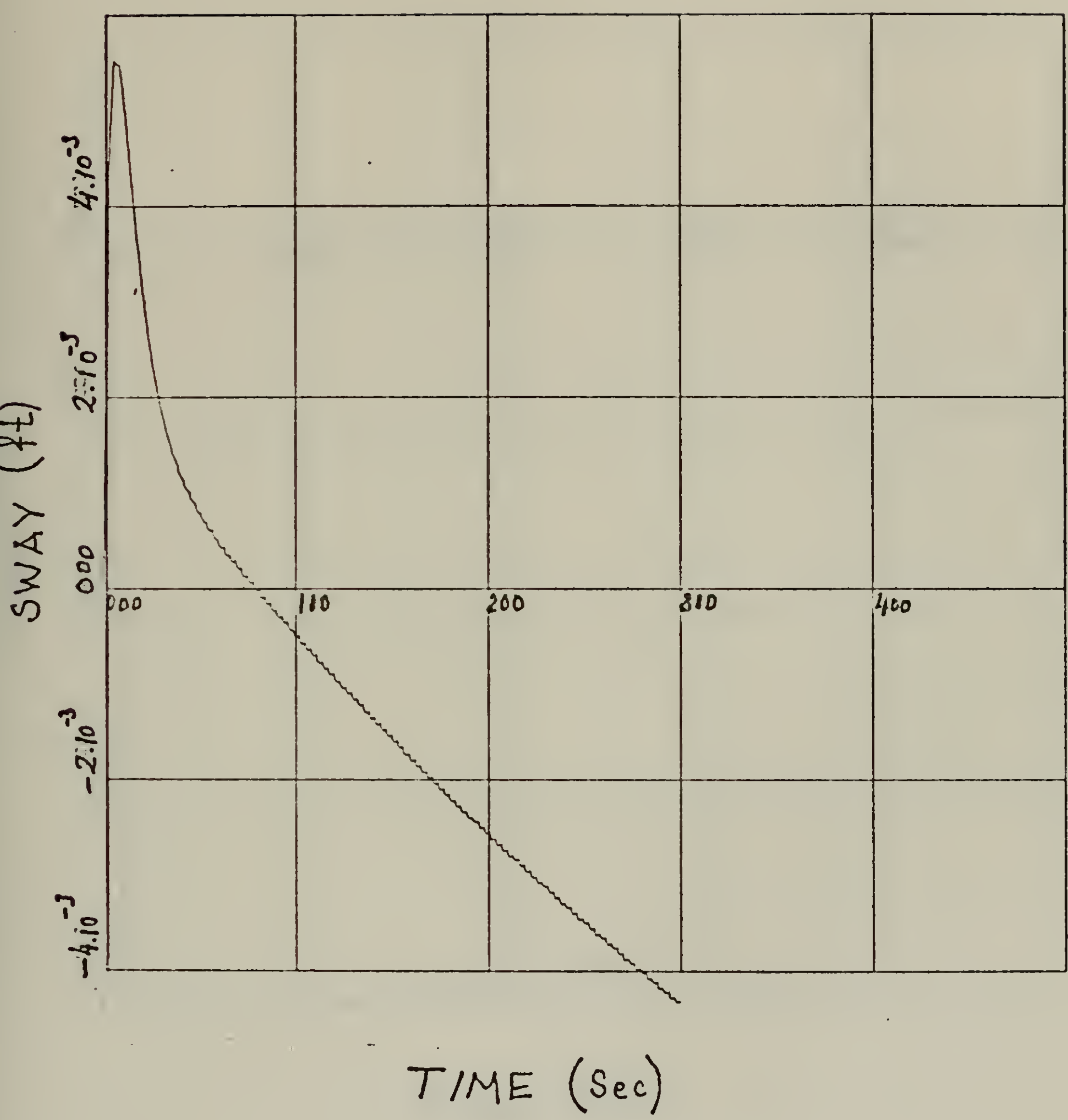


Figure 20. Variation of Sway Versus Time for Wind Direction 180°, Velocity 3 miles/hr.

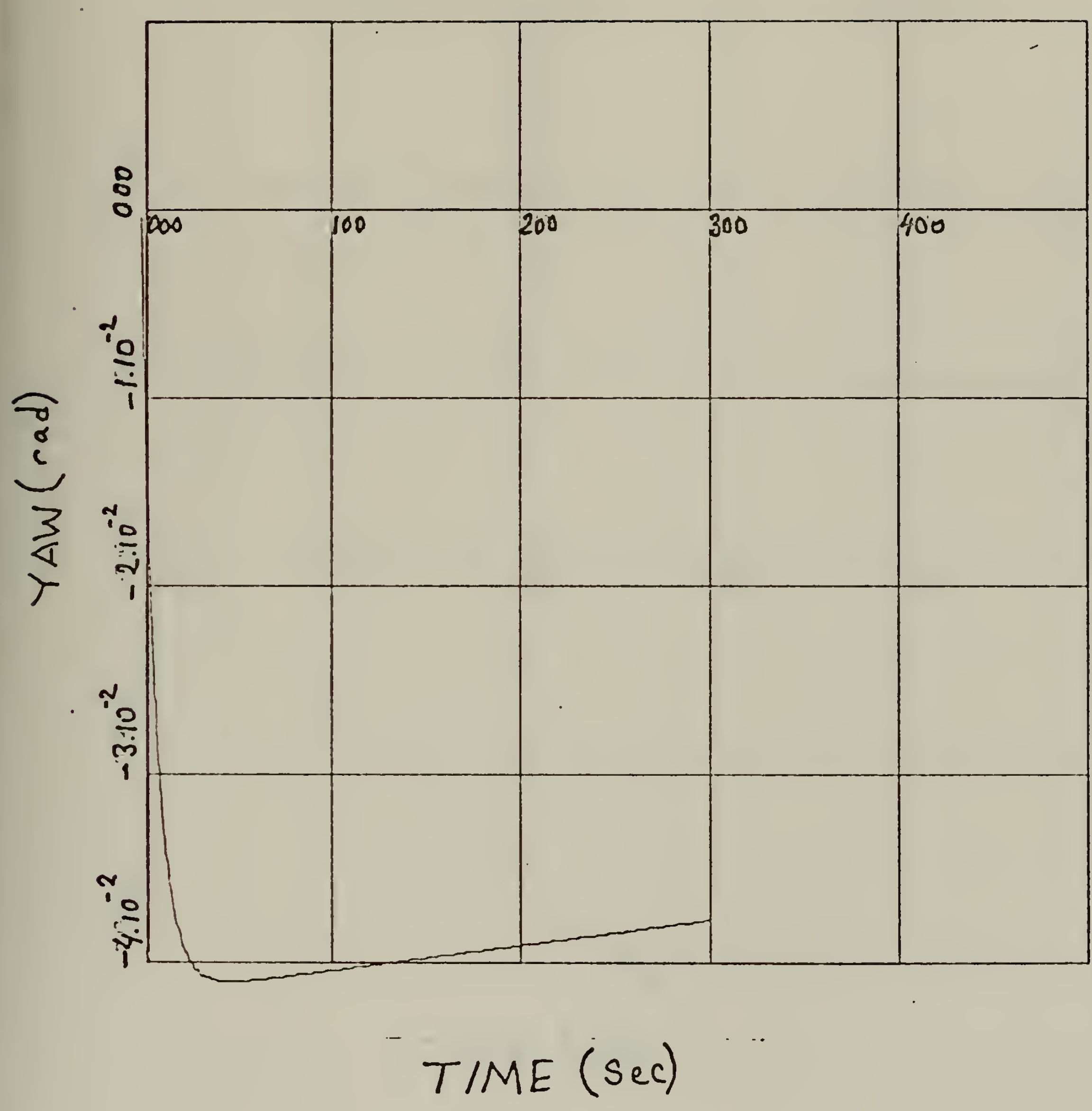


Figure 21. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 5 miles/hr.

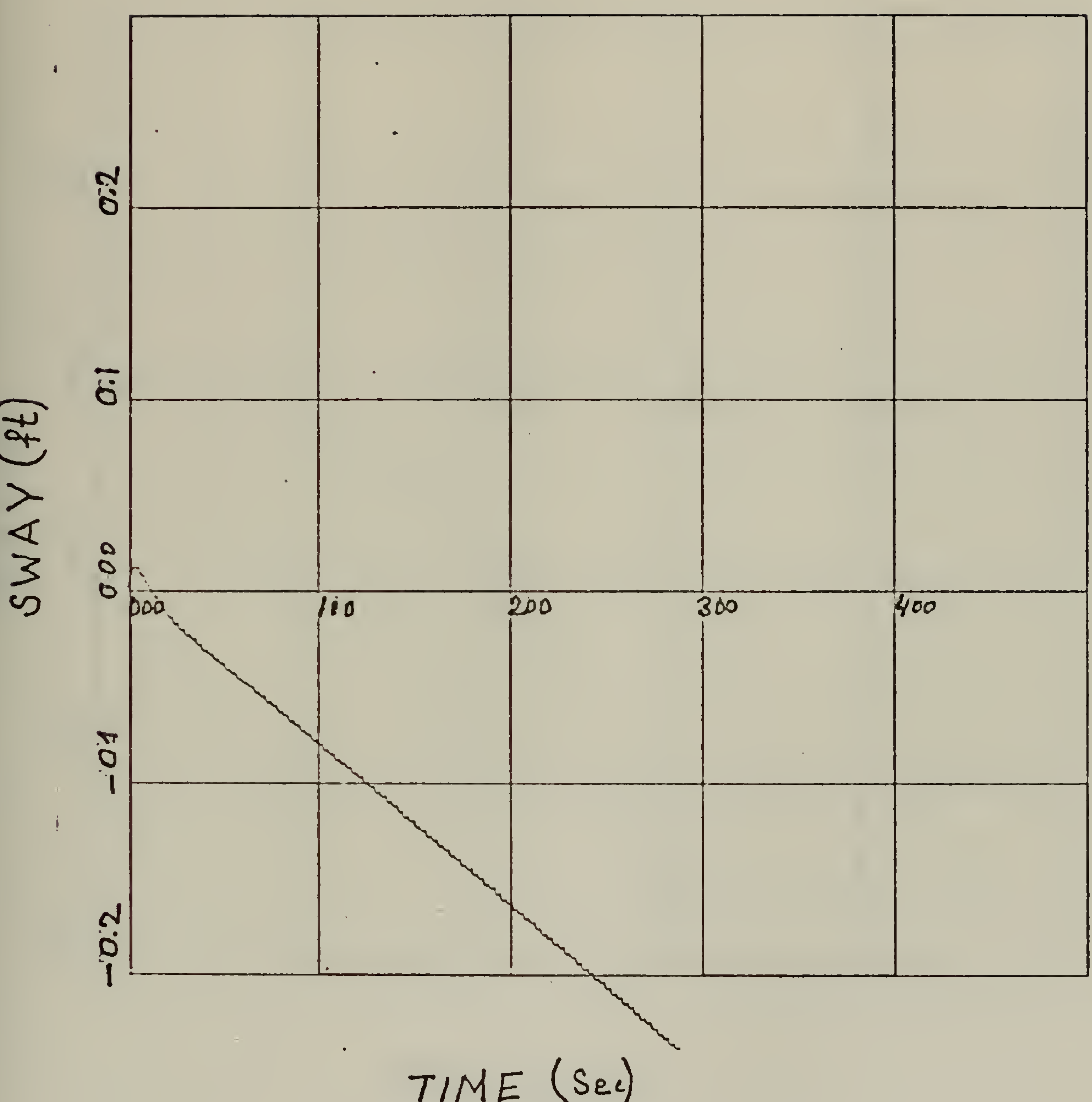


Figure 22. Variation of Sway Versus Time for Wind Direction 180°, Velocity 5 miles/hr.

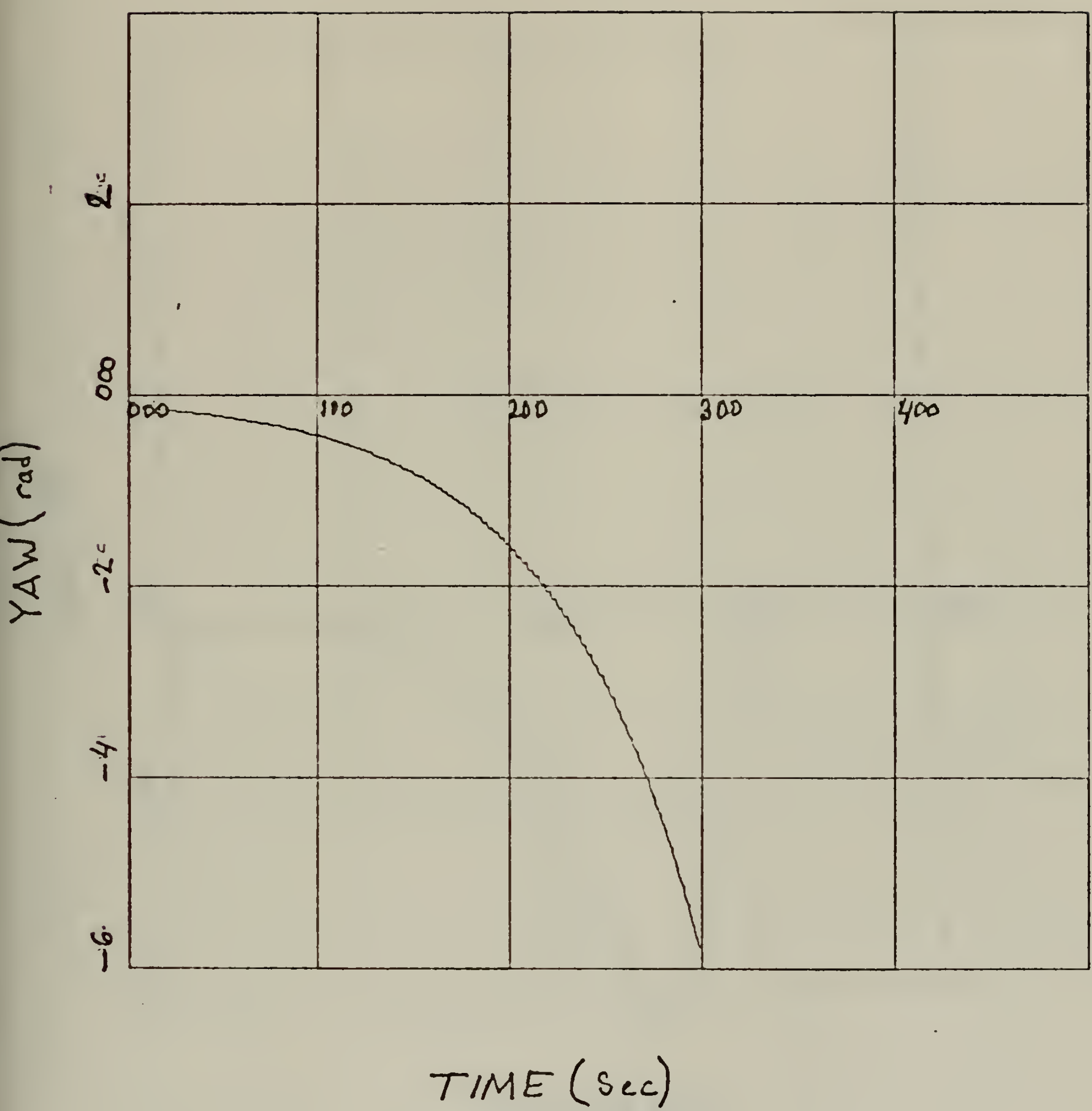


Figure 23. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 7 miles/hr.

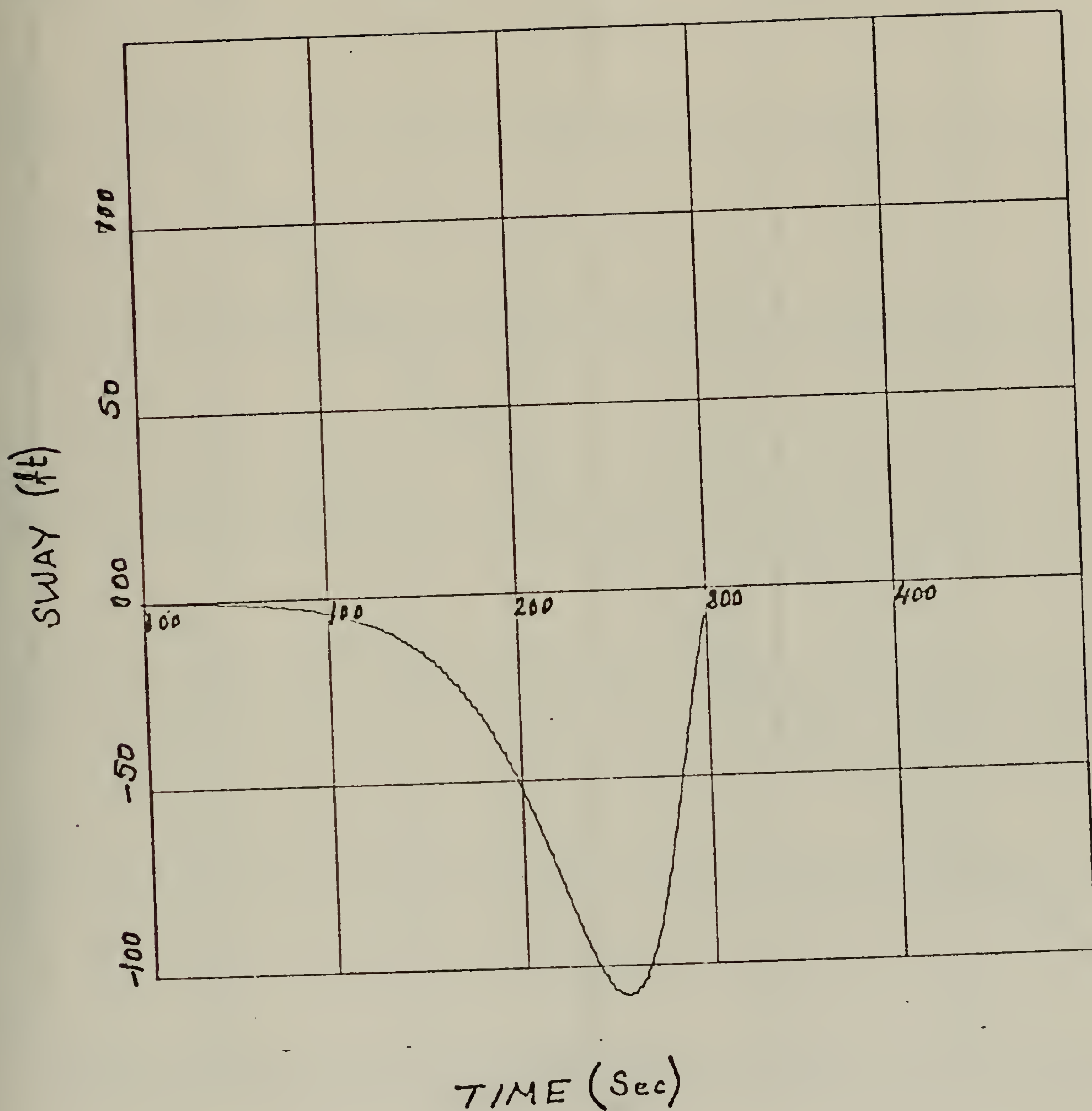


Figure 24. Variation of Sway Versus Time for Wind Direction 180°, Velocity 7 miles/hr.

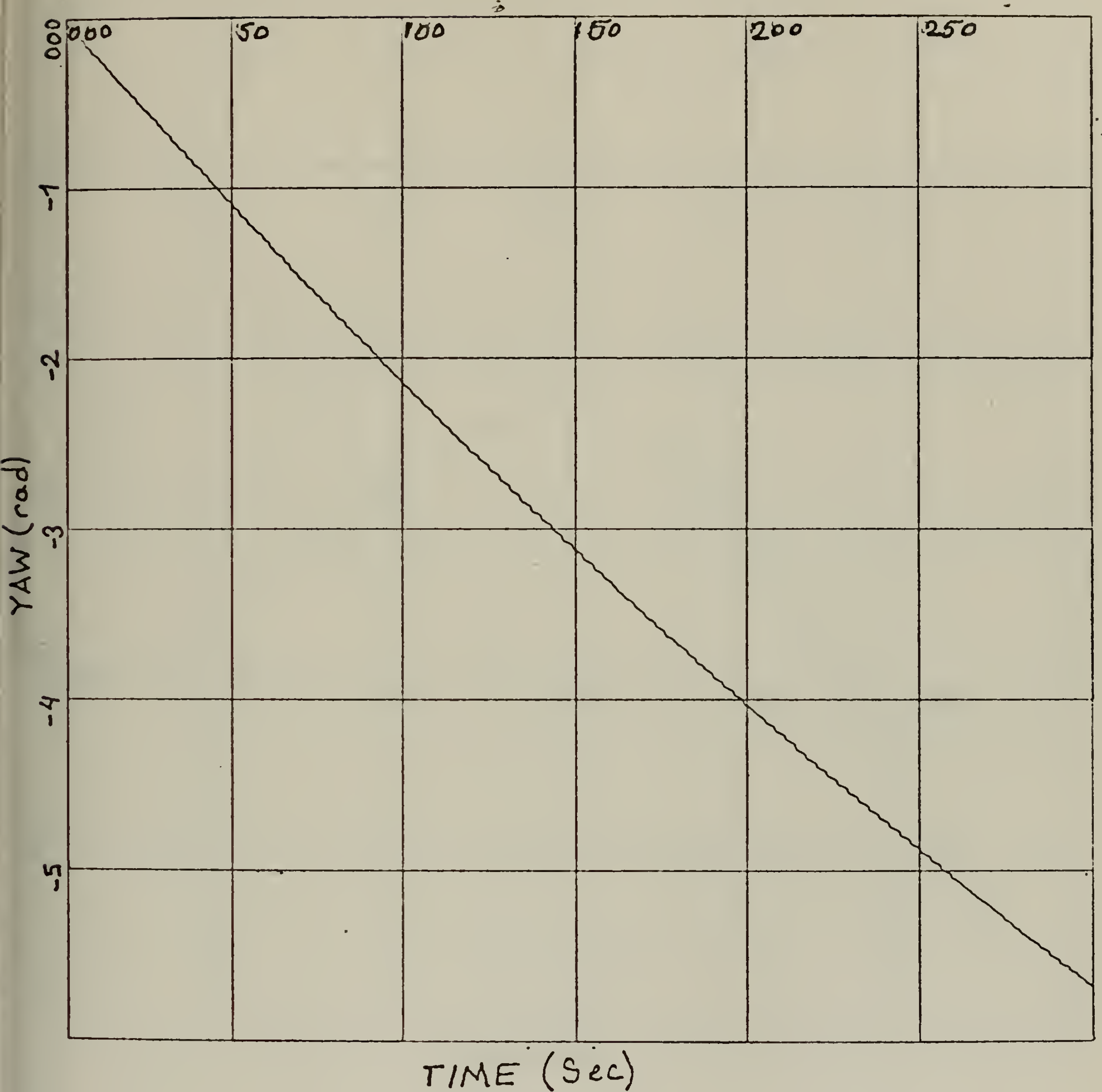


Figure 25. Variation of Yaw Versus Time for Wind Direction 090°, Velocity 5 miles/hr, Rudder Deflection 15°.

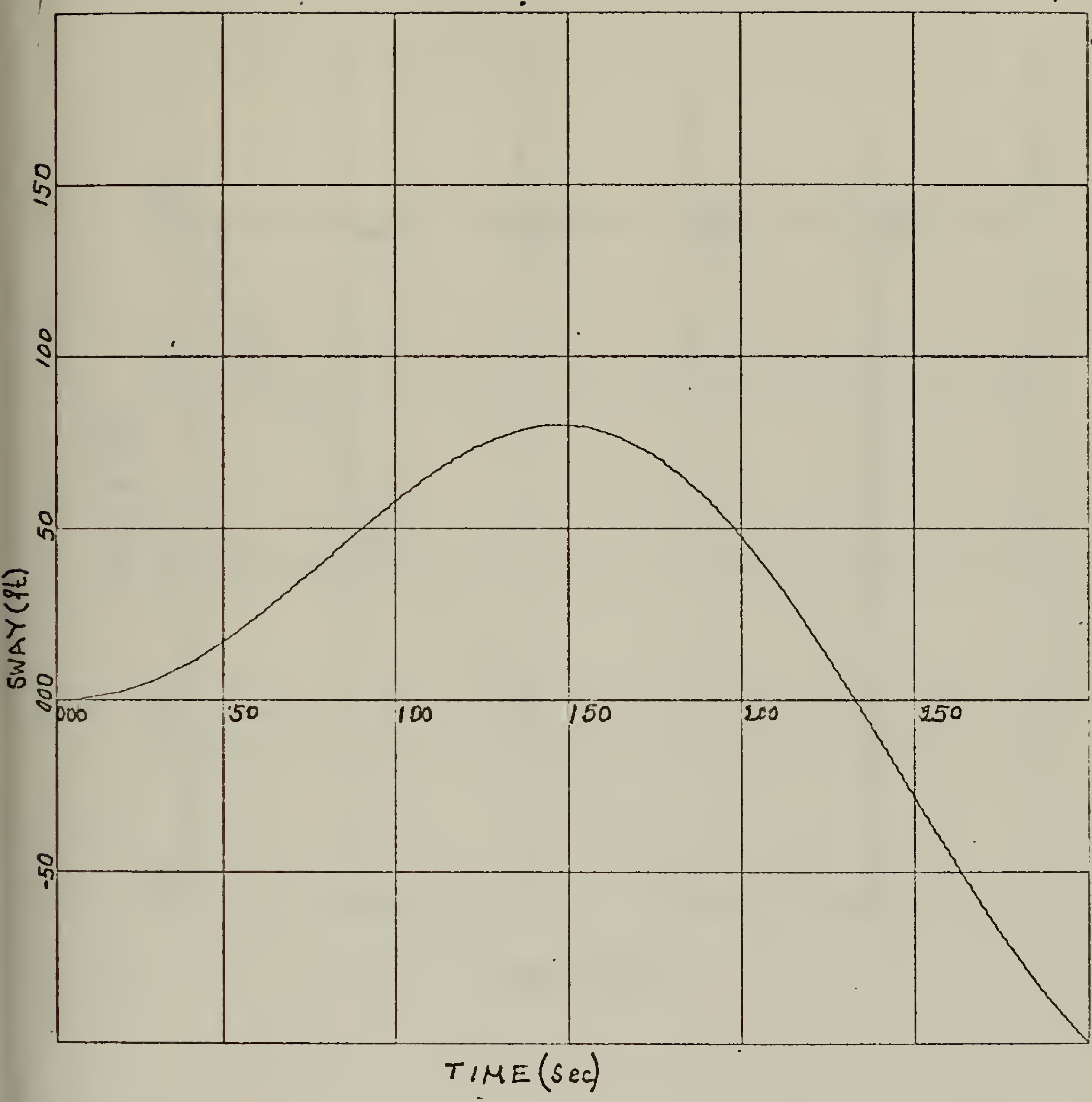


Figure 26. Variation of Sway Versus Time for Wind Direction 090°, Velocity 5 miles/hr, Rudder Deflection 15°.

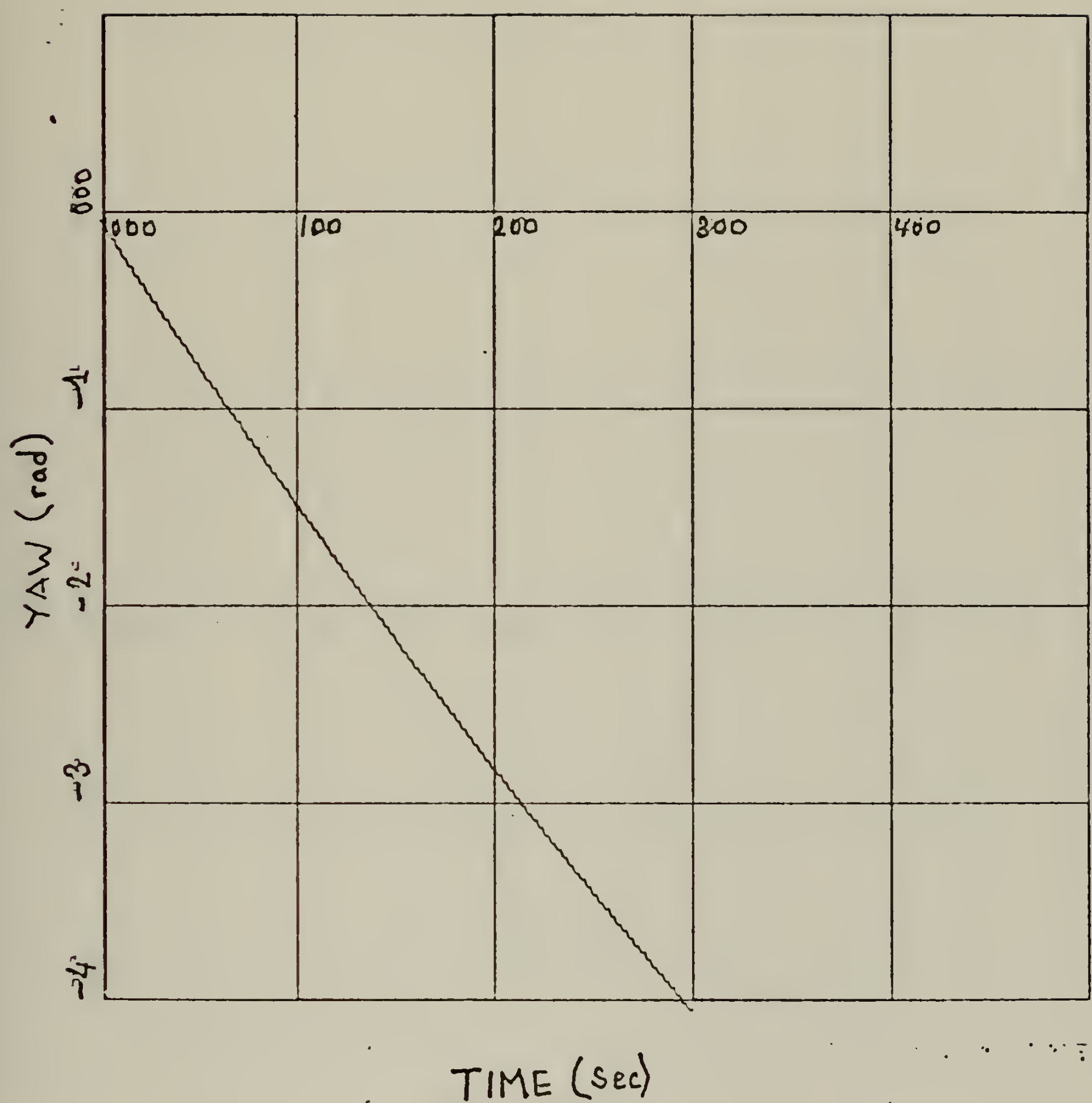


Figure 27. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 3 miles/hr, Rudder Deflection 15°.

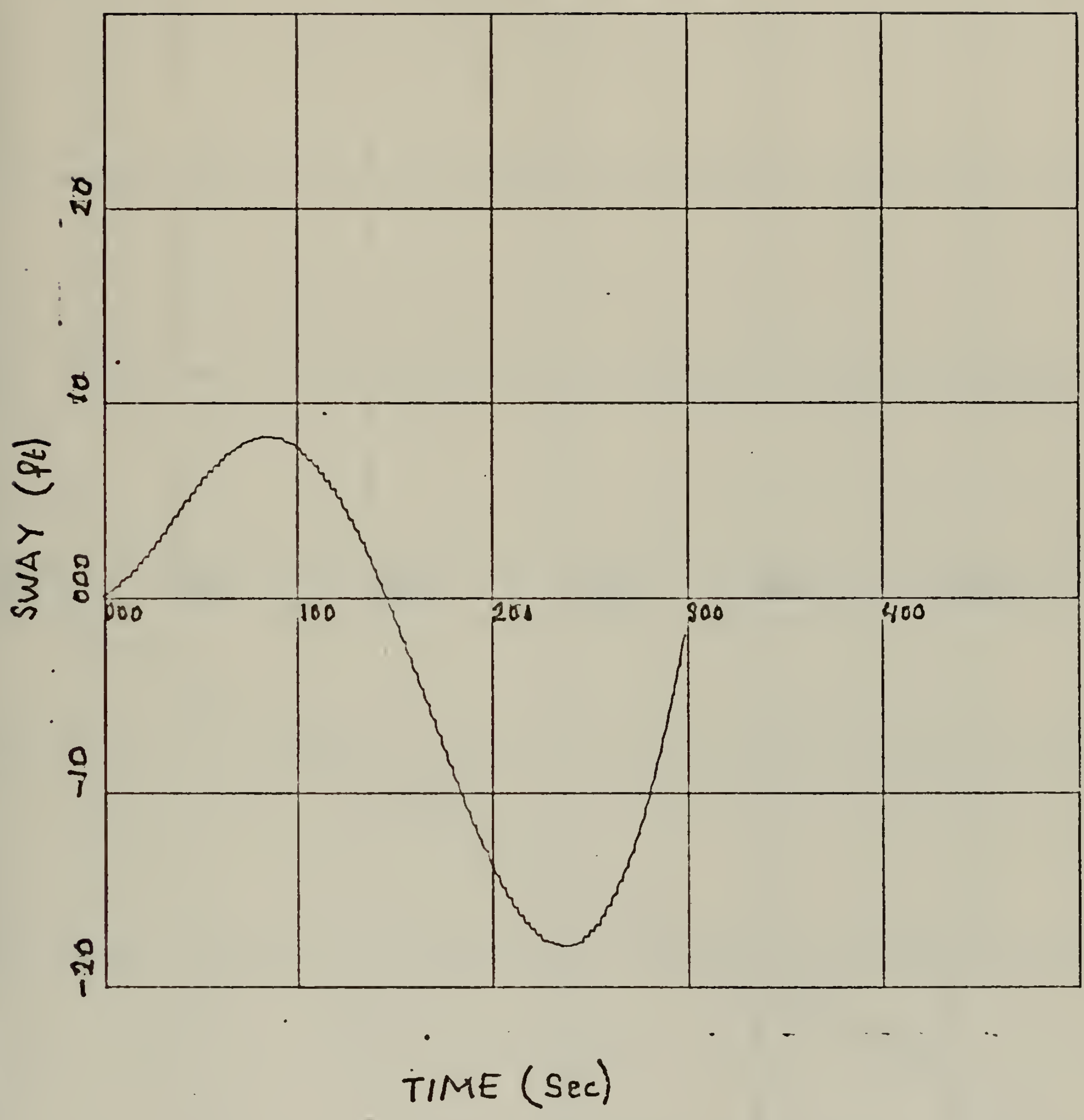


Figure 28. Variation of Sway Versus Time for Wind Direction 180°, Velocity 3 miles/hr, Rudder Deflection 15°.

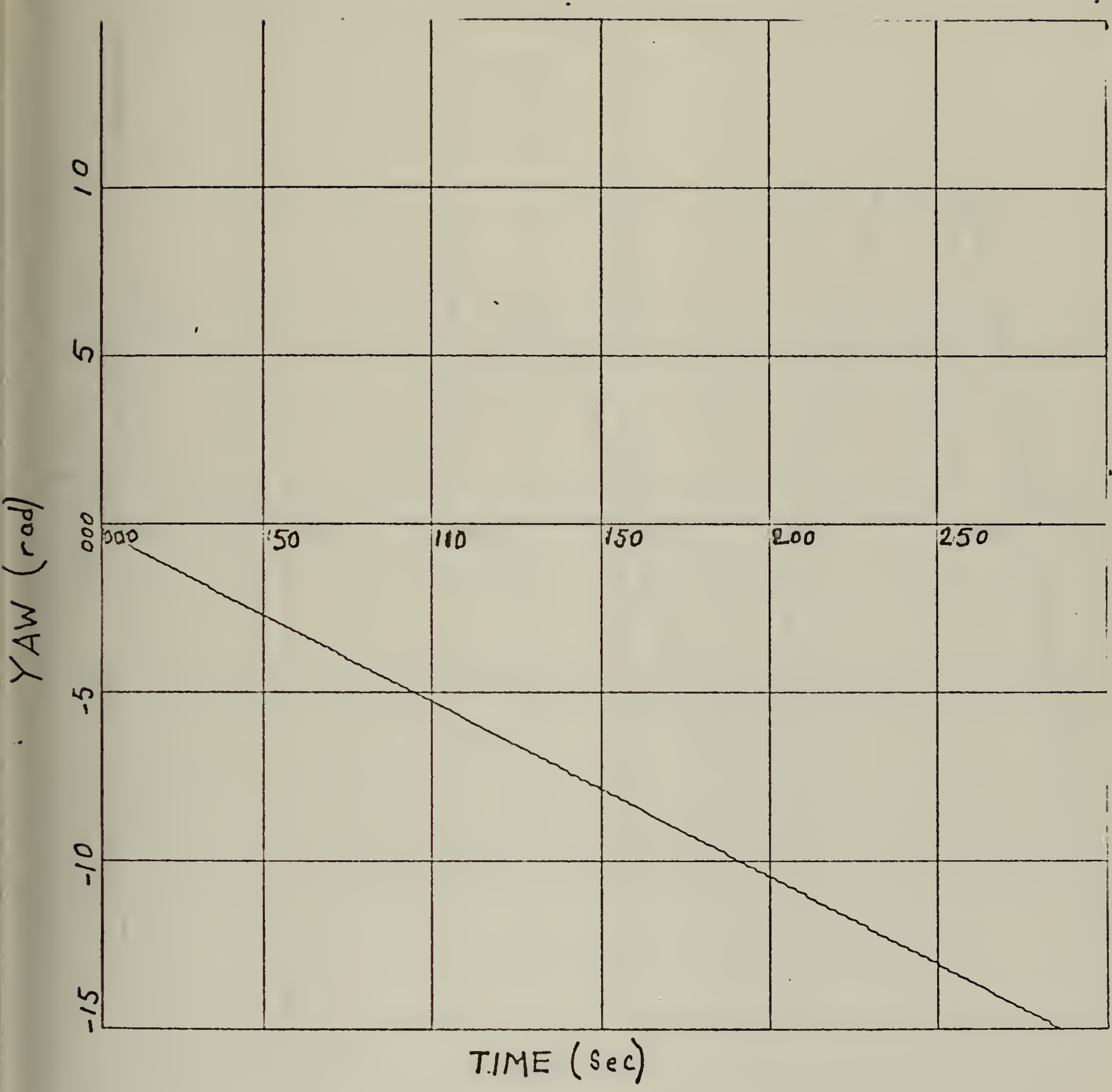


Figure 29. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 5 miles/hr, Rudder Deflection 15°.

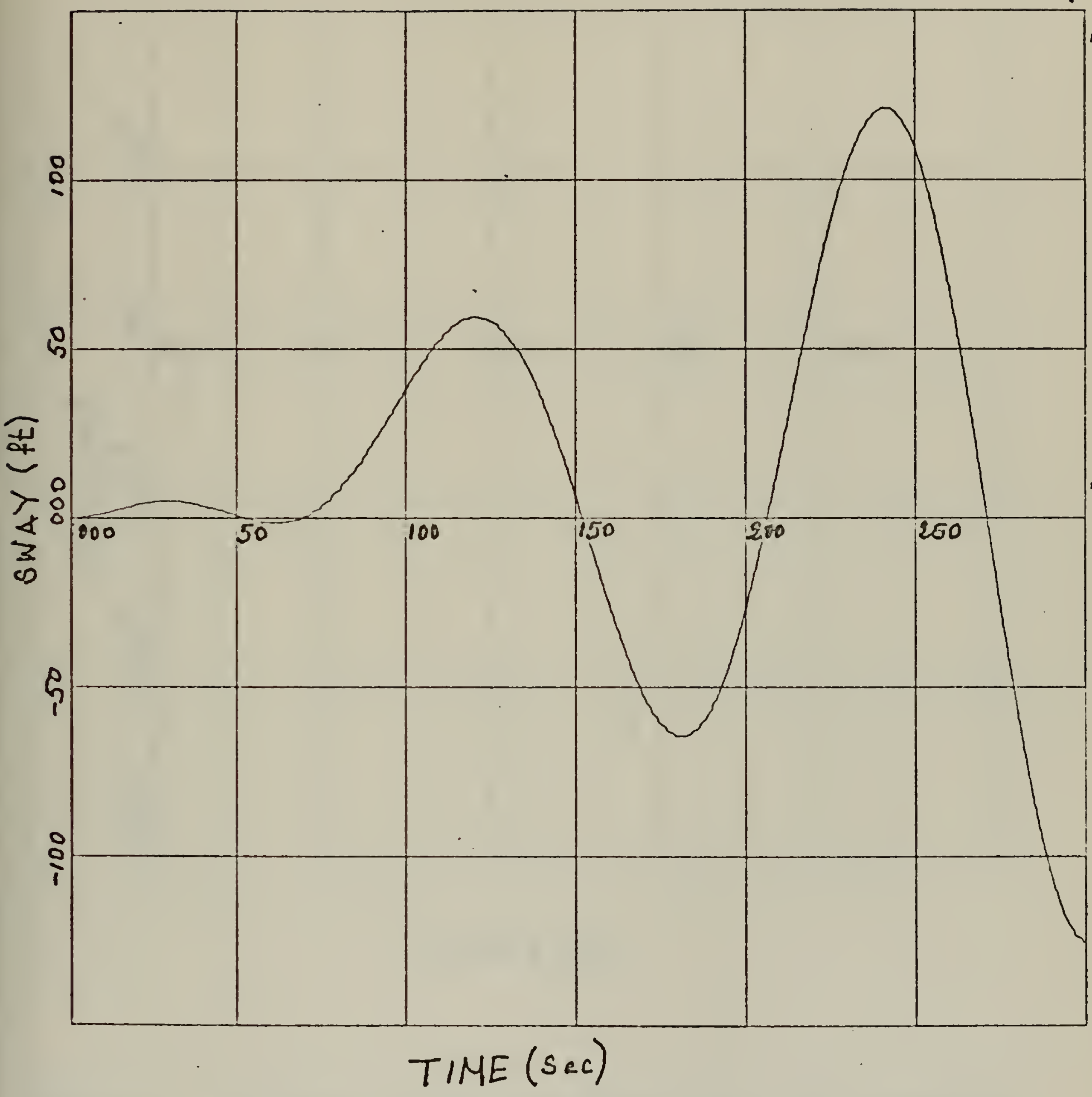


Figure 30. Variation of Sway Versus Time for Wind Direction 180°, Velocity 5 miles/hr, Rudder Deflection 15°.

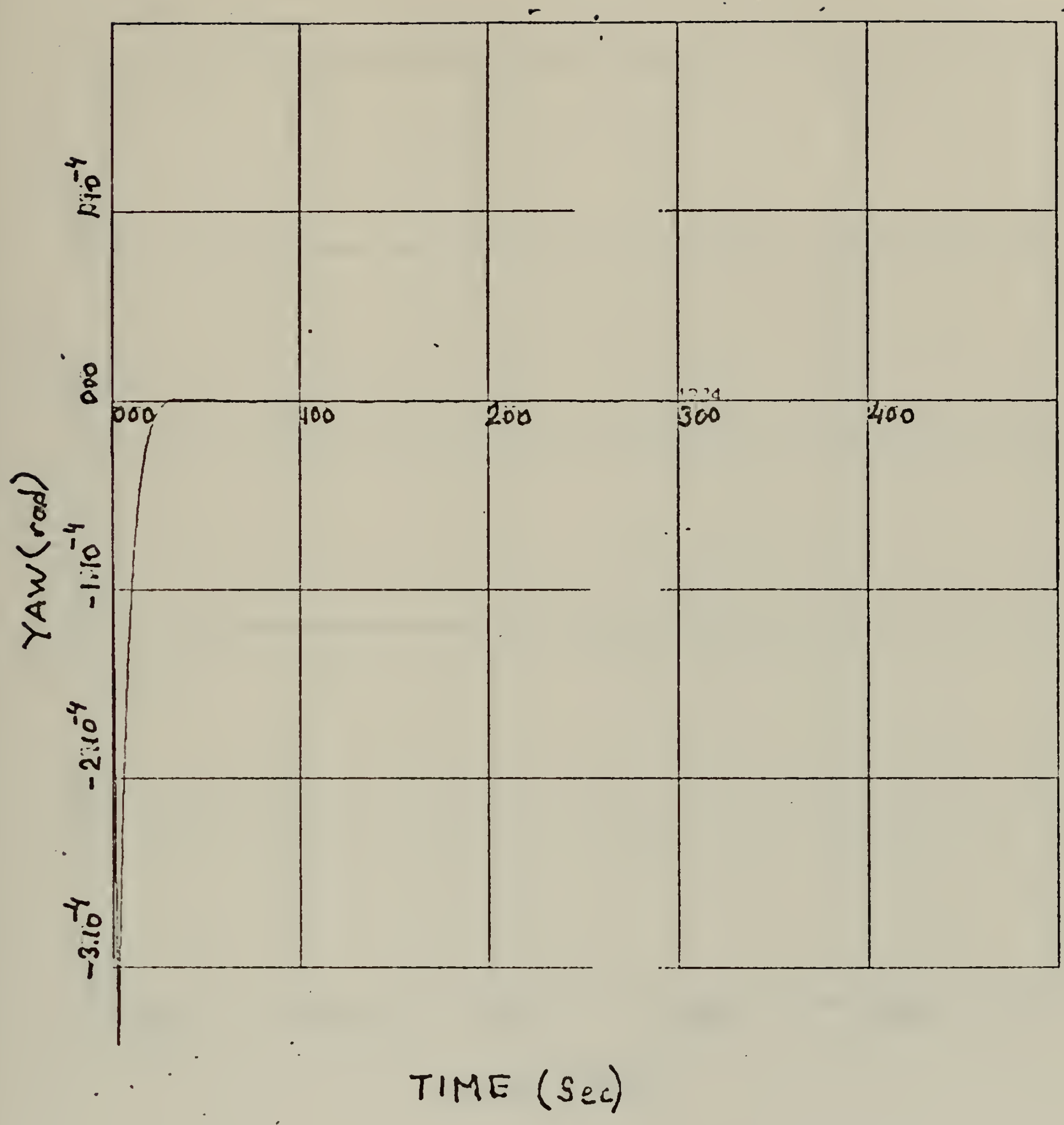


Figure 31. Variation of Yaw Versus Time for Wind Direction 000°, Velocity 1 mile/hr, Automatically Steered.

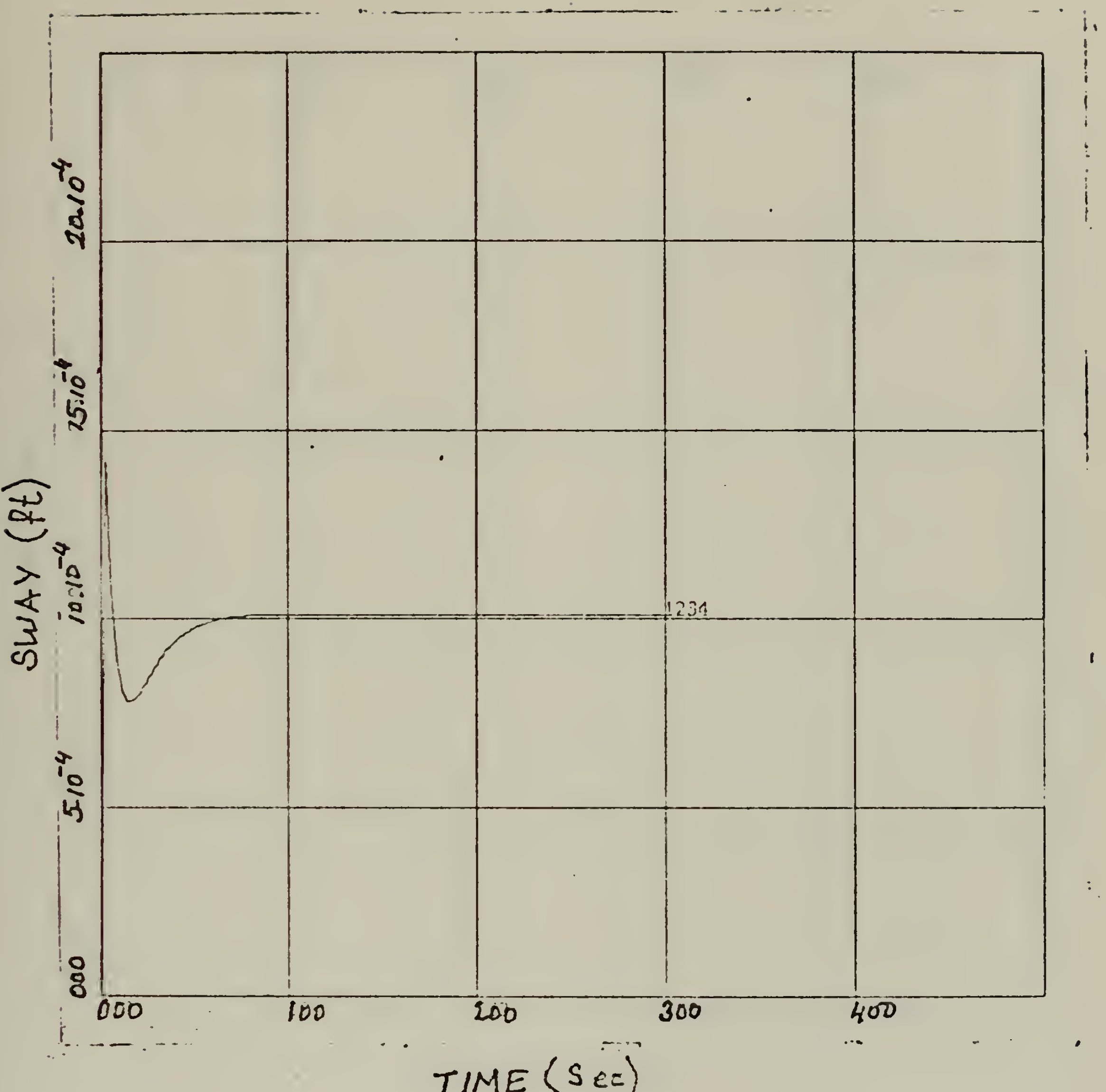


Figure 32. Variation of Sway Versus Time for Wind Direction 000°, Velocity 1 mile/hr, Automatically Steered.

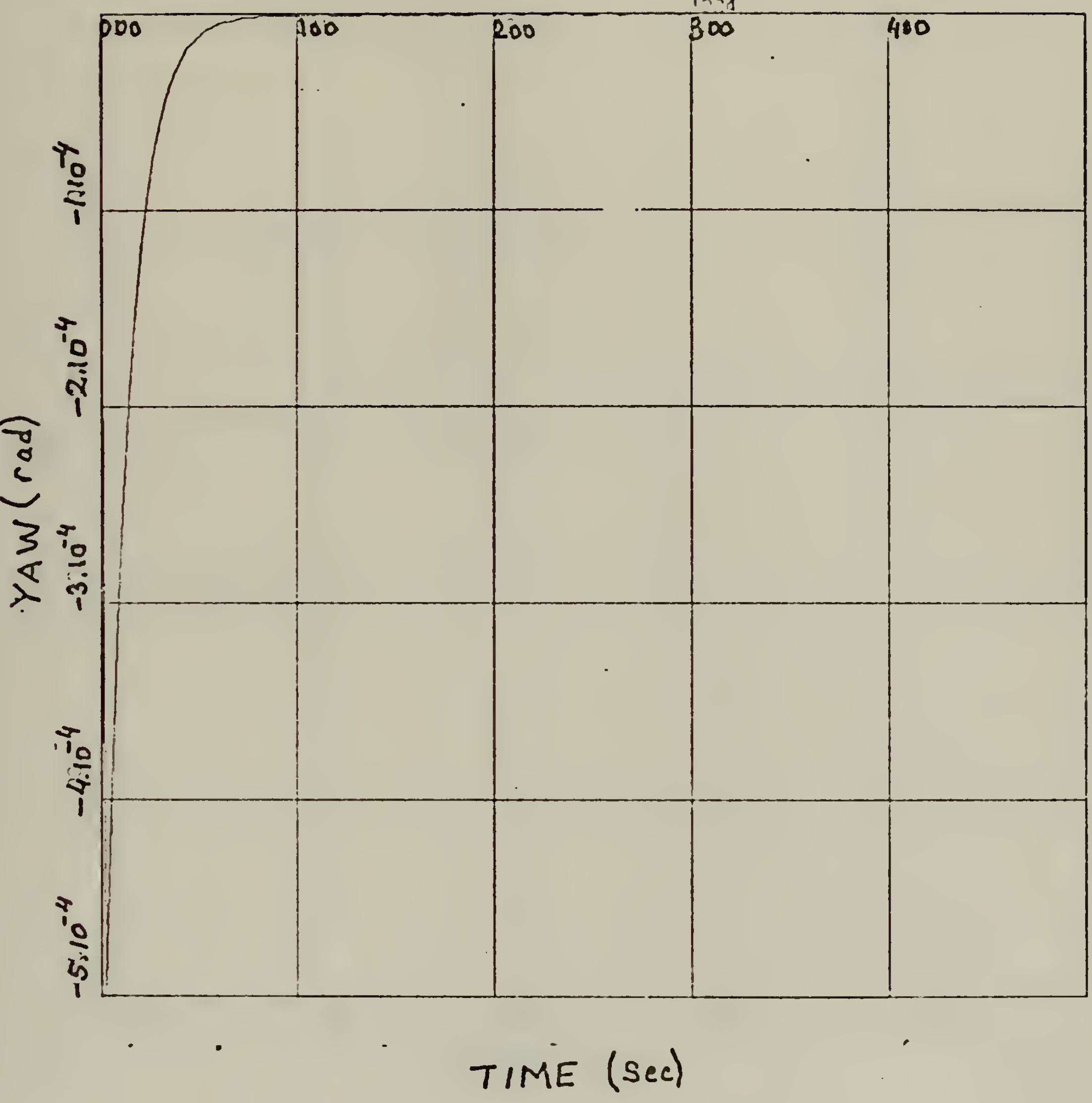


Figure 33. Variation of Yaw Versus Time for Wind Direction 000°, Velocity 3 miles/hr, Automatically Steered.

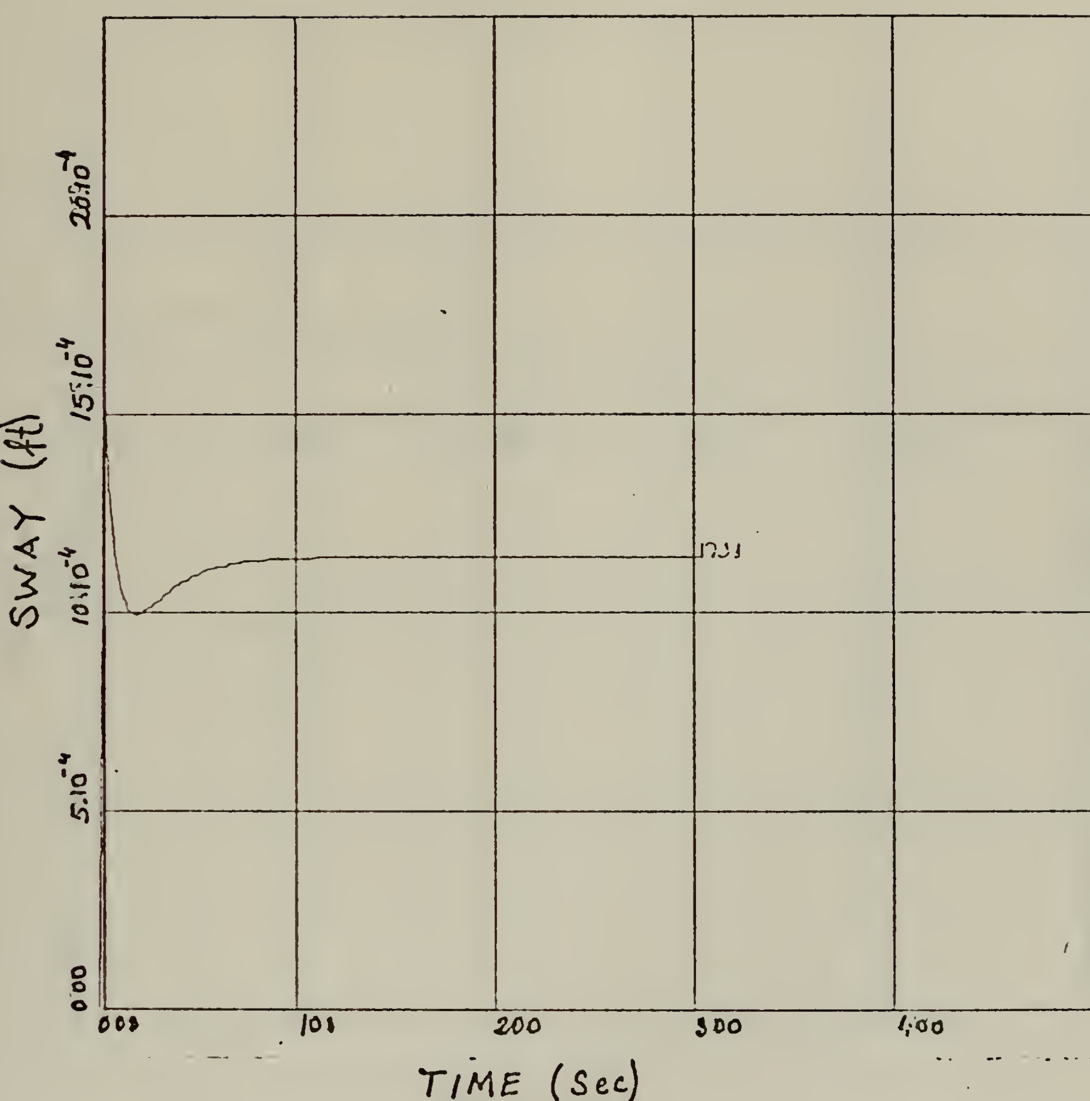


Figure 34. Variation of Sway Versus Time for Wind Direction 000°, Velocity 3 miles/hr, Automatically Steered.

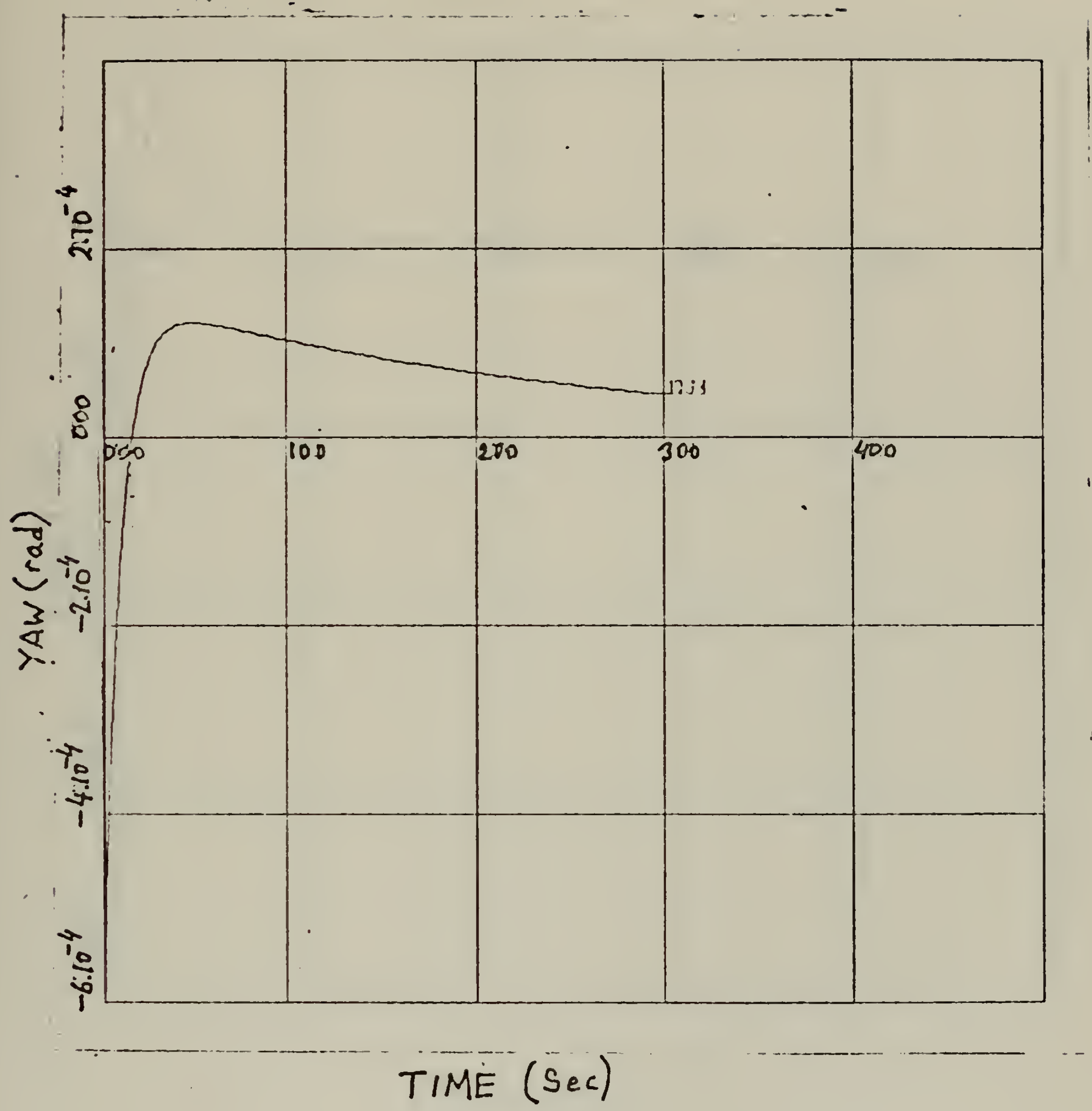


Figure 35. Variation of Yaw Versus Time for Wind Direction 000°, Velocity 5 miles/hr, Automatically Steered.

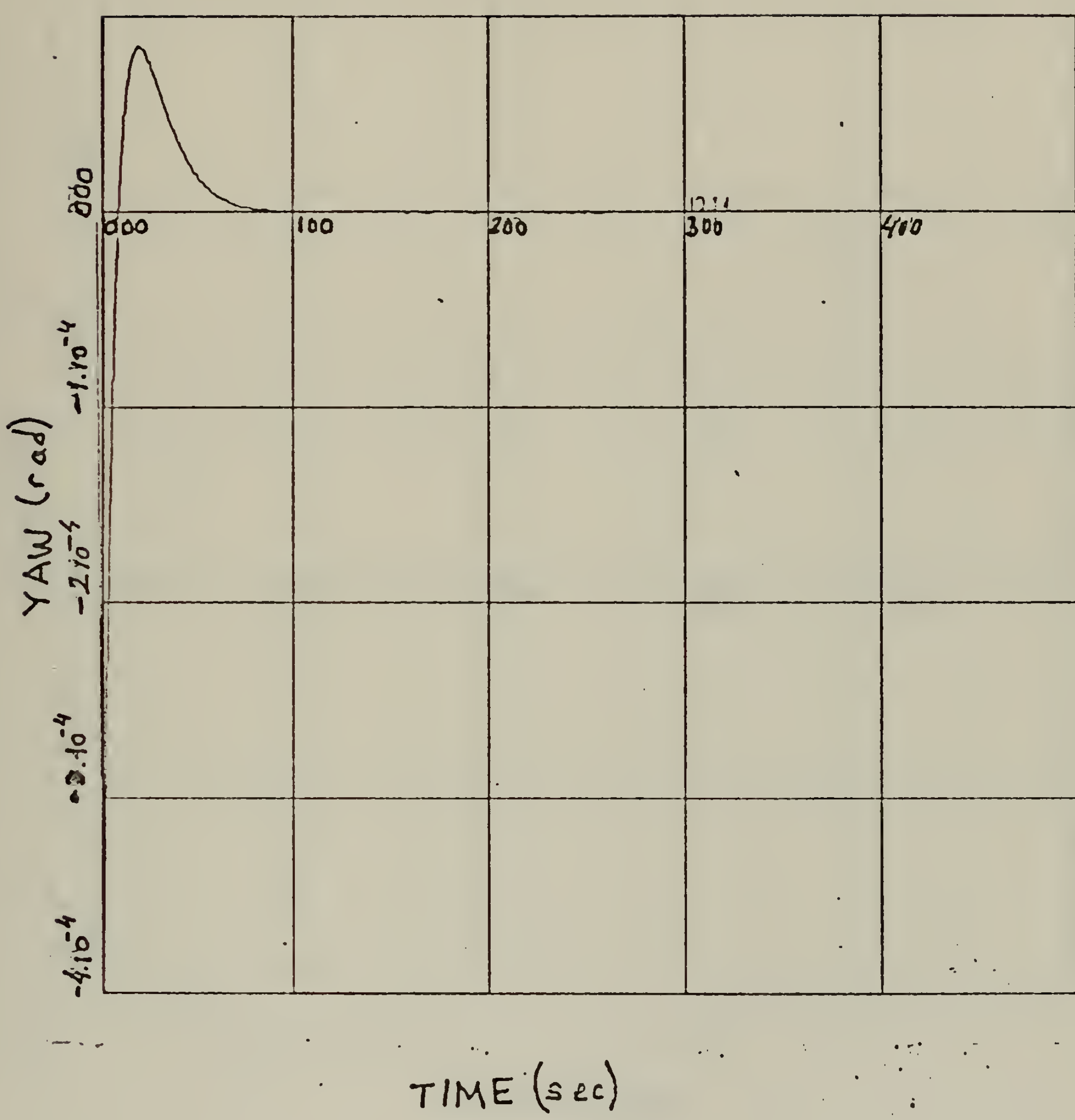


Figure 36. Variation of Yaw Versus Time for Wind Direction 090°, Velocity 3 miles/hr, Automatically Steered.

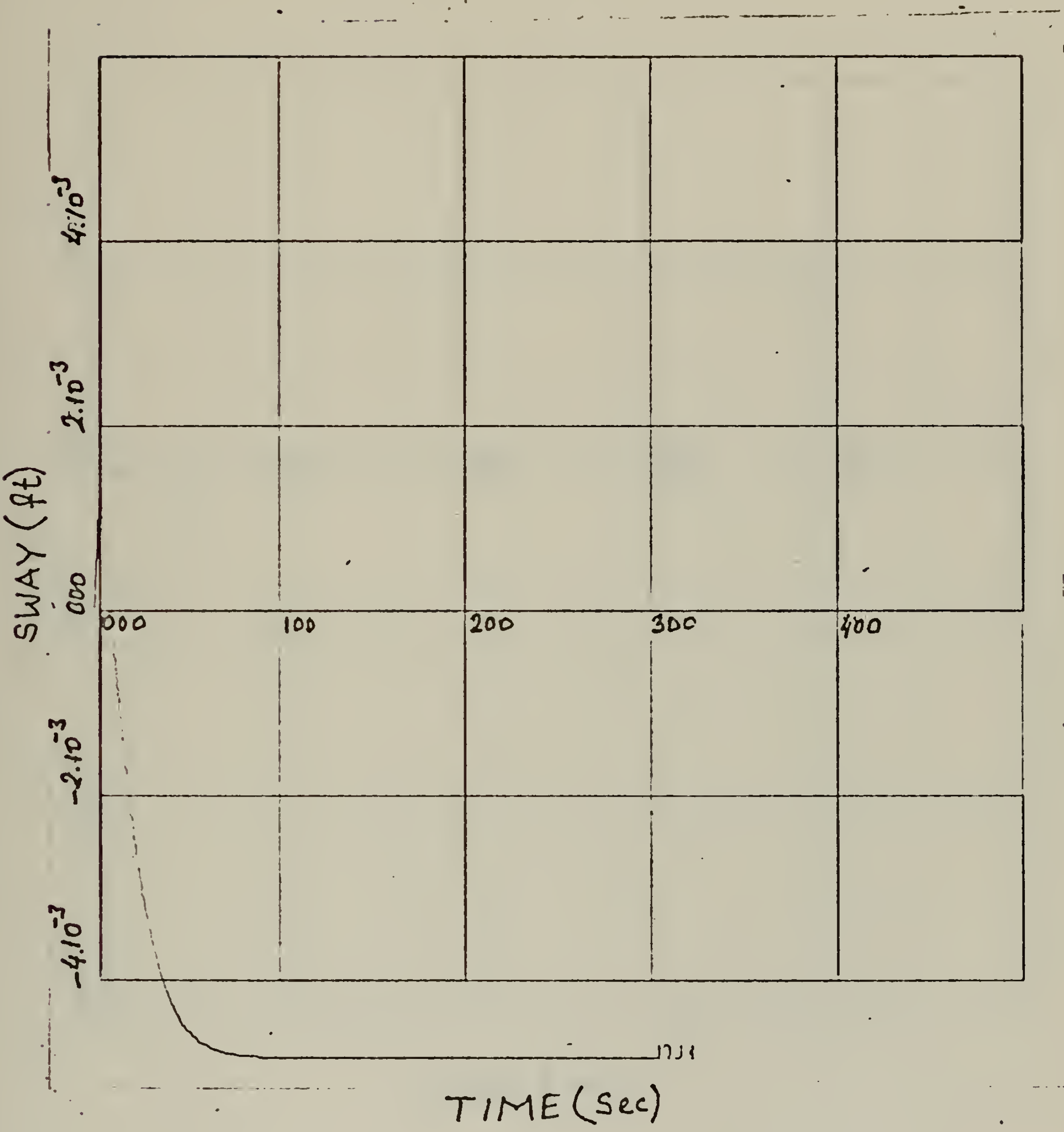


Figure 37. Variation of Sway Versus Time for Wind Direction 090°, Velocity 3 miles/hr, Automatically Steered.

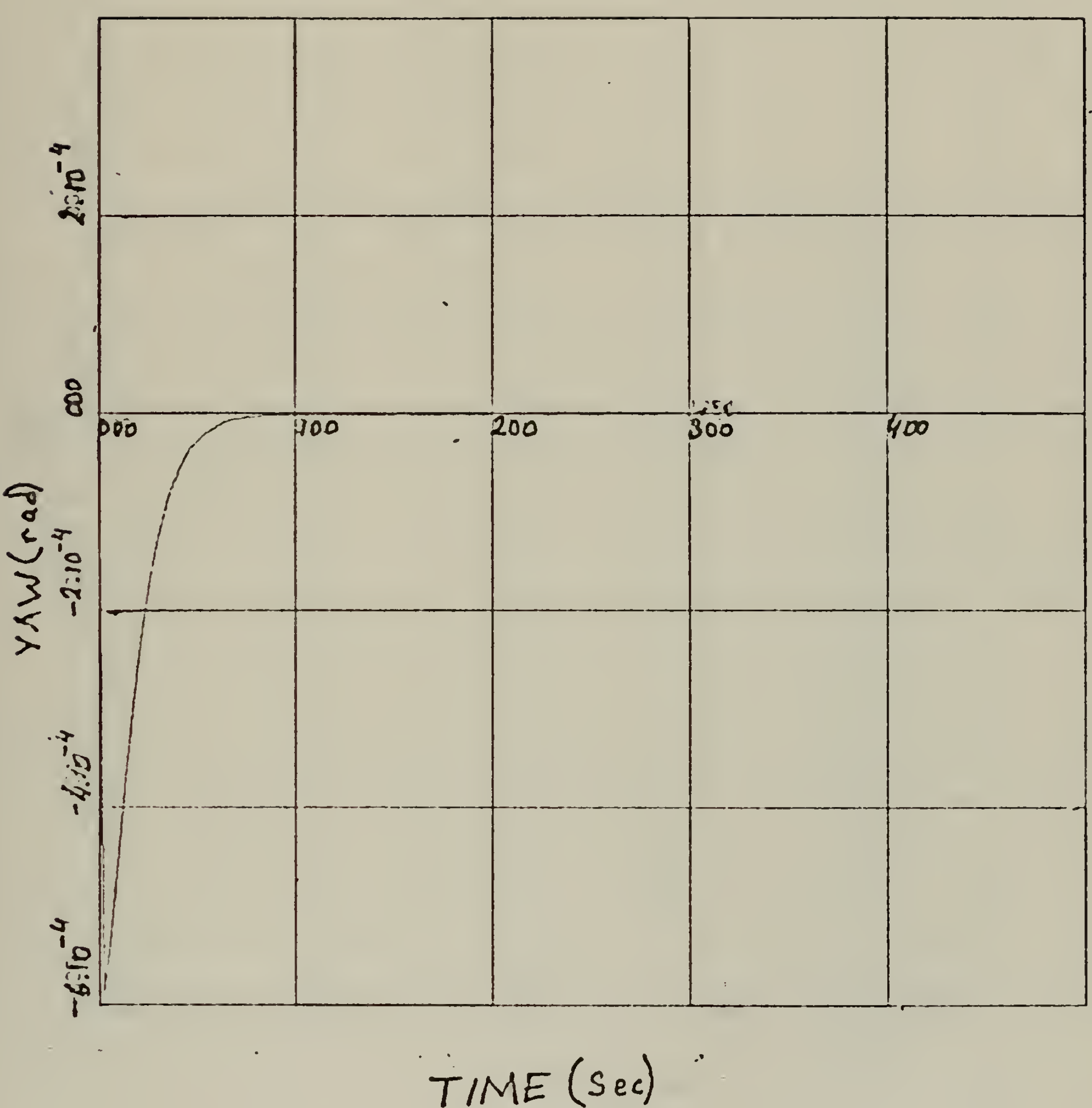


Figure 38. Variation of Yaw Versus Time for Wind Direction 180°, Velocity 1 mile/hr, Automatically Steered.

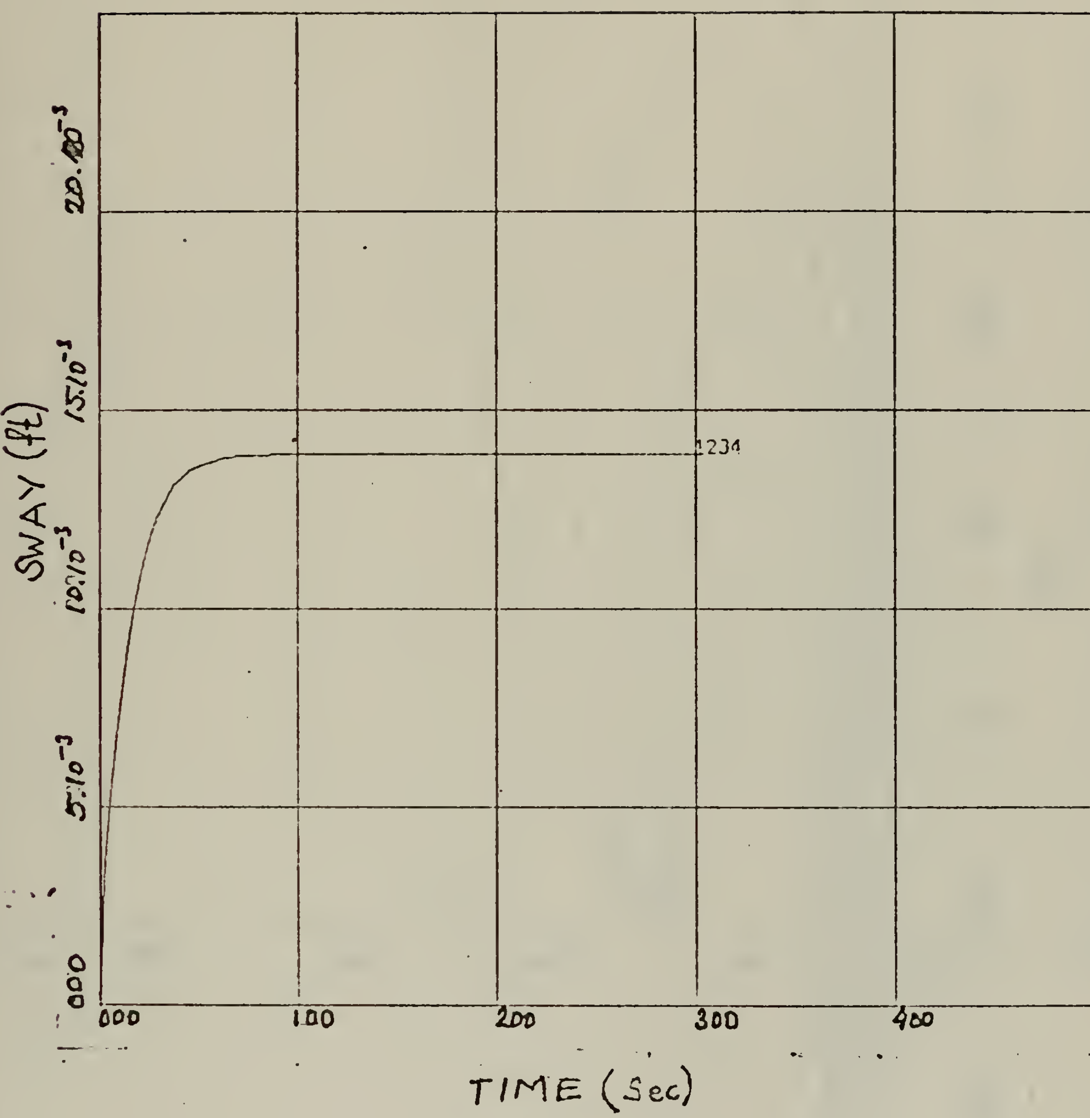


Figure 39. Variation of Sway Versus Time for Wind Direction 180°, Velocity 1 mile/hr, Automatically Steered.

Figure 40. Yaw Gain Constant Versus Yaw Rate Gain Constant for Wind Direction 000°, Velocity 1 mile/hr.

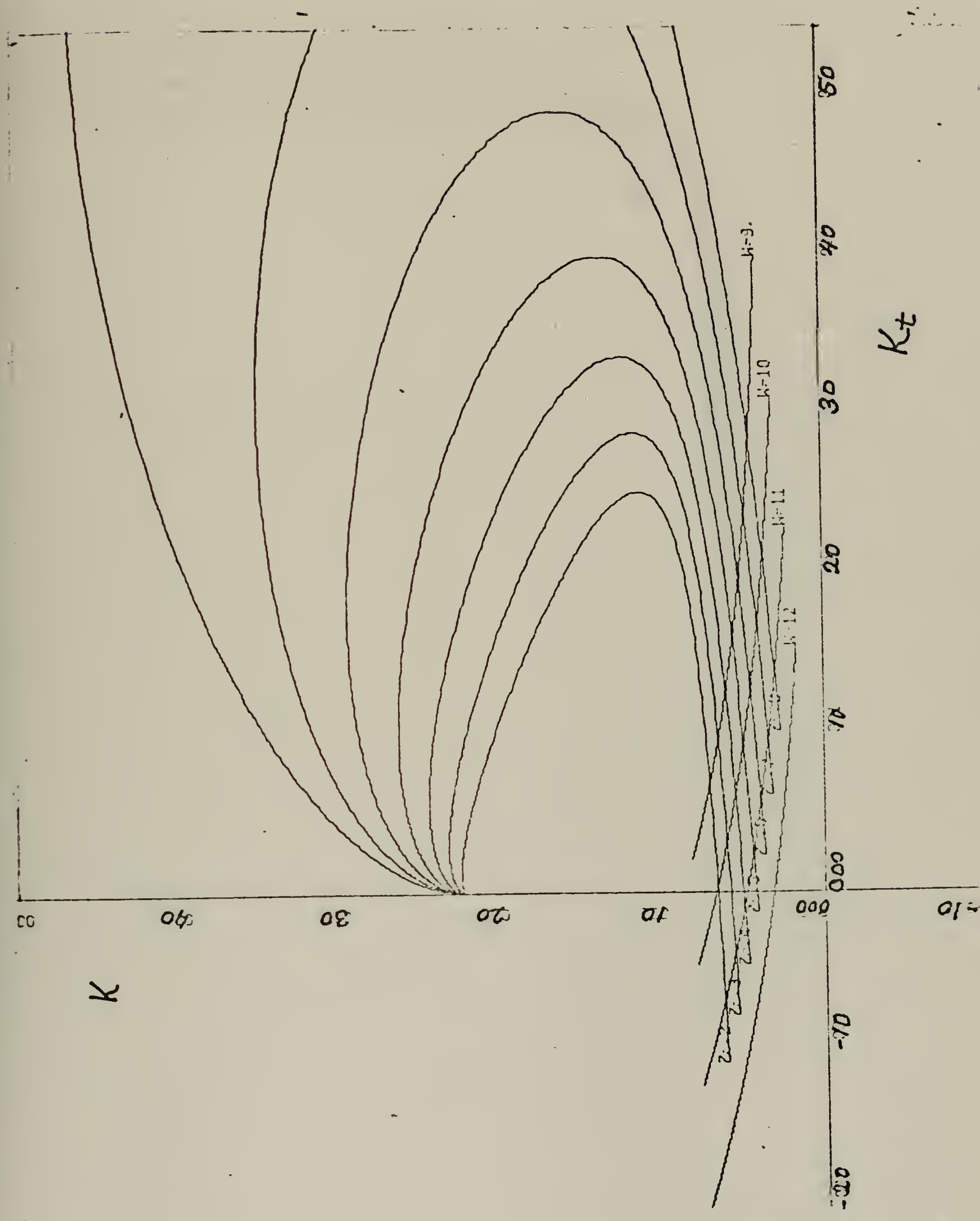


Figure 41. Yaw Gain Constant Versus Yaw Rate Gain Constant for Wind Direction 000°, Velocity 3 miles/hr.

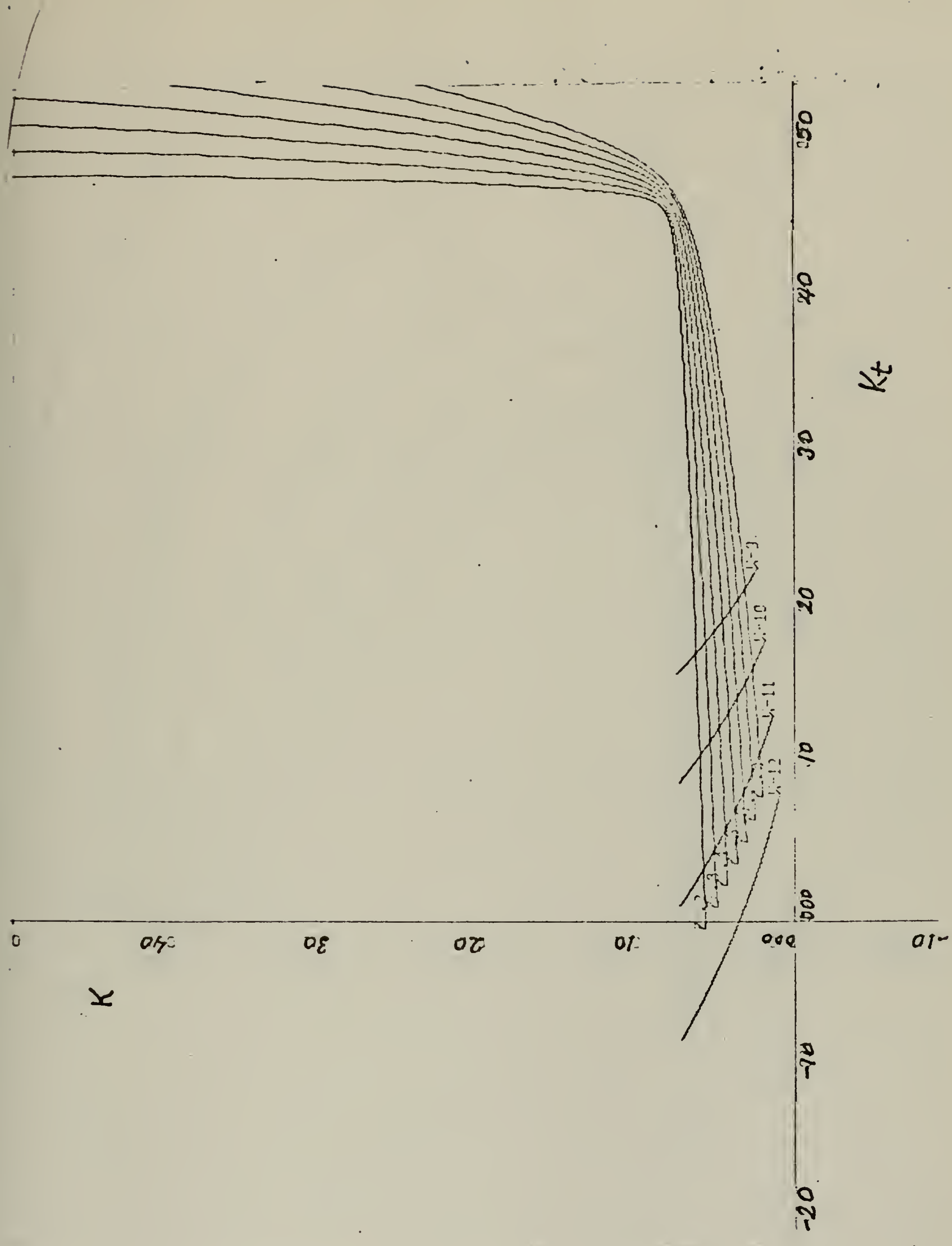


Figure 42. Yaw Gain Constant Versus Yaw Rate Gain Constant for Wind Direction 000°, Velocity 5 miles/hr.

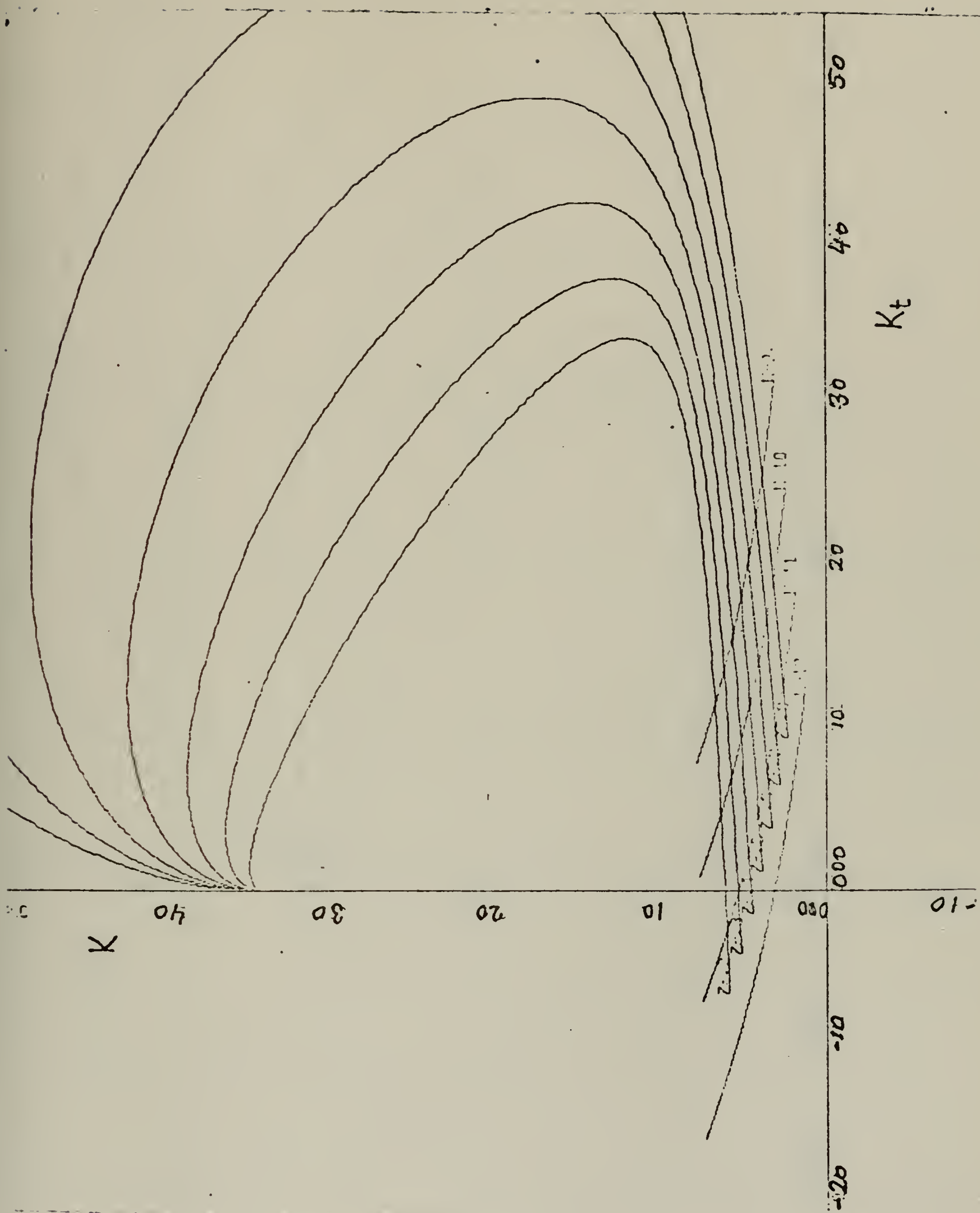


Figure 43. Yaw Gain Constant Versus Yaw Rate Gain Constant for Wind Direction 090°, Velocity 3 miles/hr.

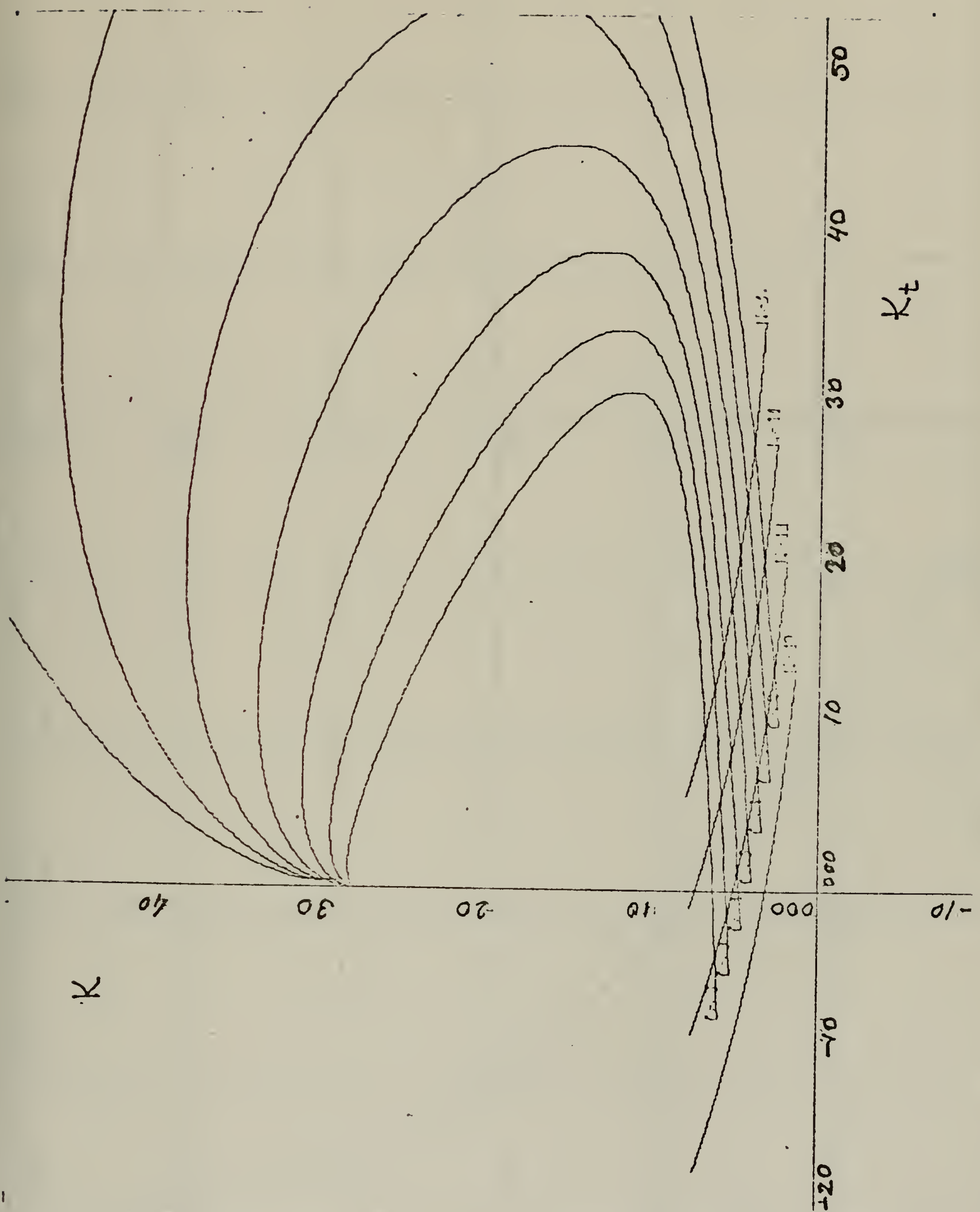


Figure 44. Yaw Gain Constant Versus Yaw Rate Gain Constant for Wind Direction 180°, Velocity 1 mile/hr.

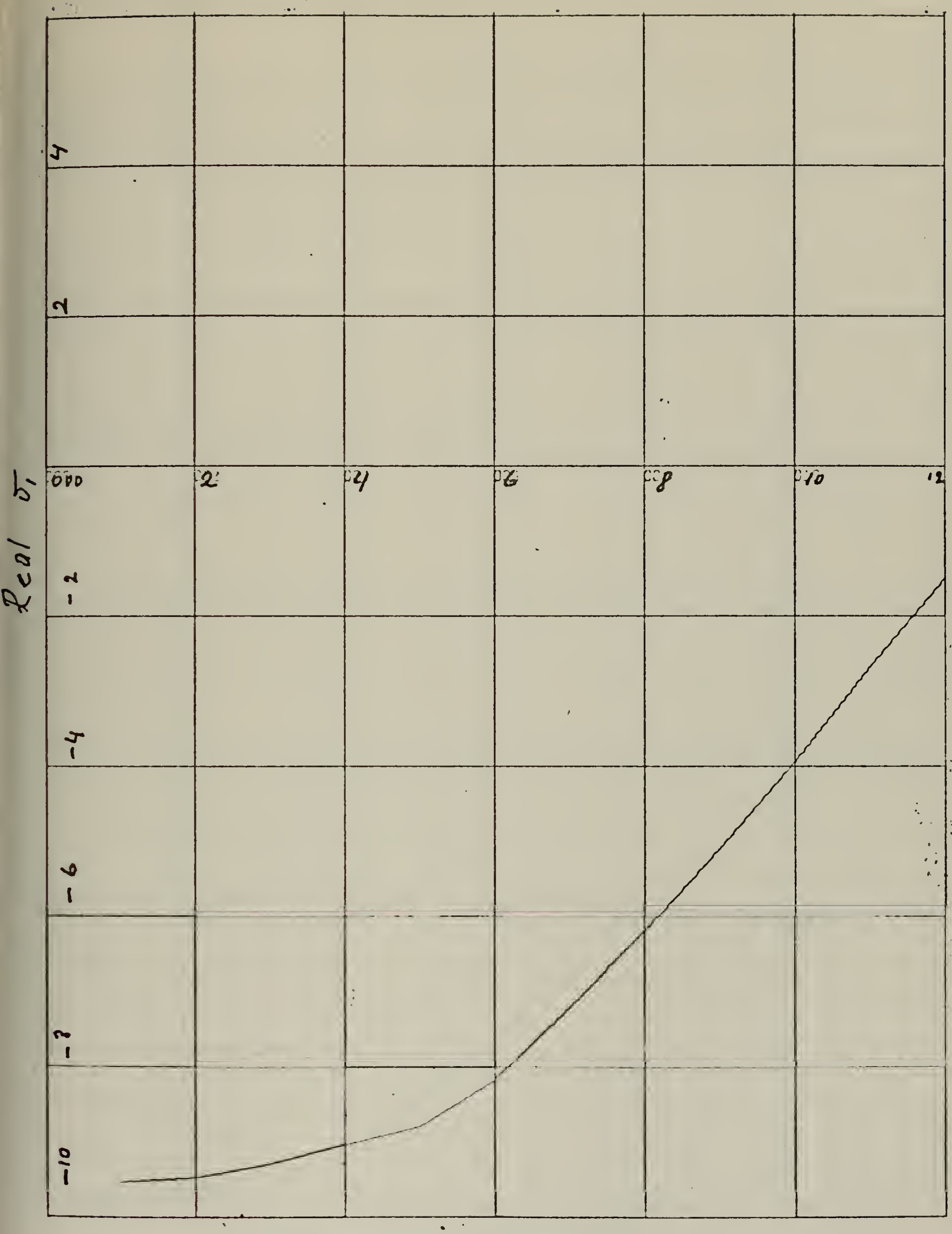


Figure 45. Real σ_1 Versus Wind Velocity for Head Wind.

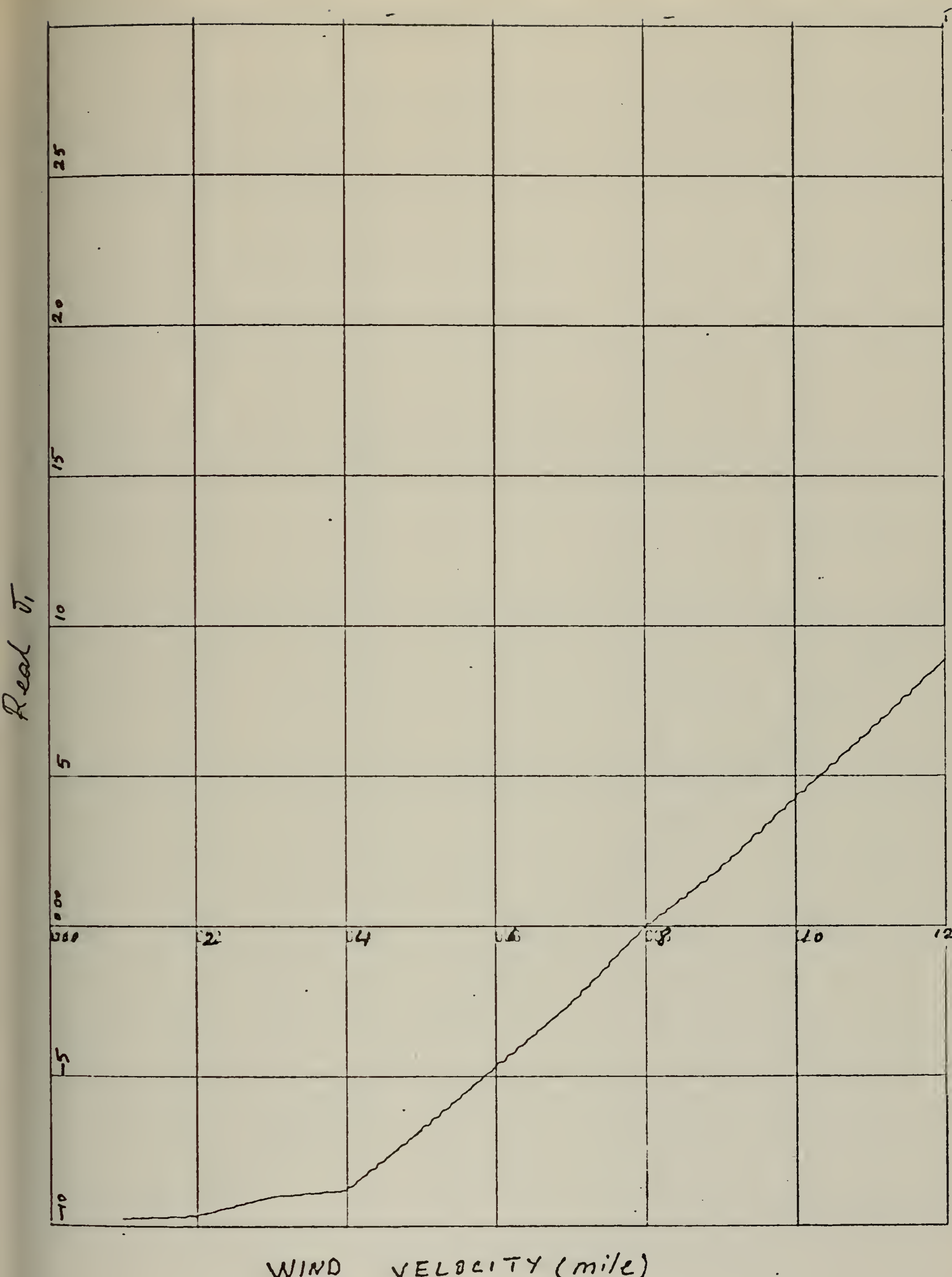


Figure 46. Real σ_1 Versus Wind Velocity for Beam Wind.

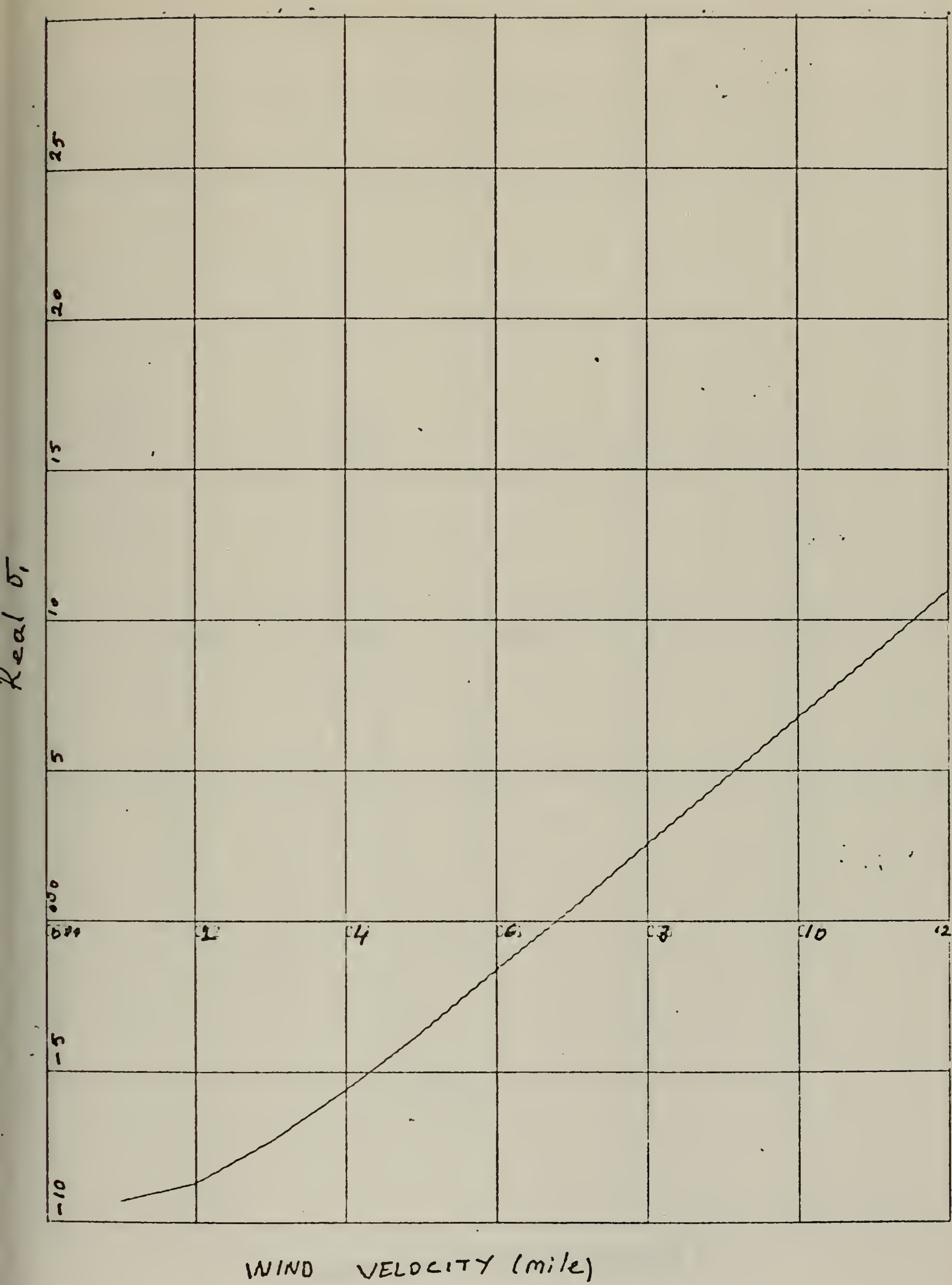


Figure 47. Real σ_1 Versus Wind Velocity for Stern Wind.

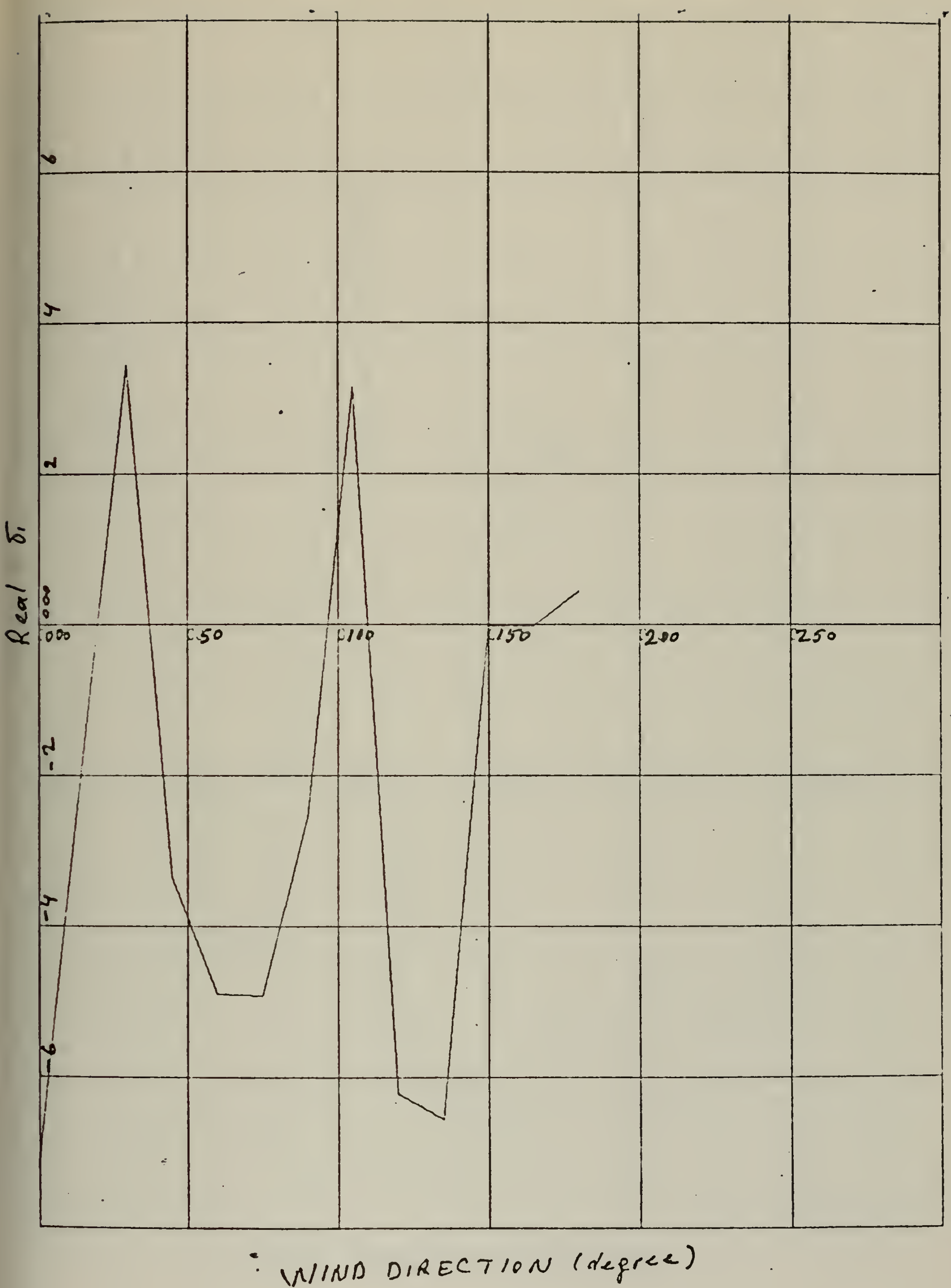


Figure 48. Real σ_1 Versus Wind Direction for 7 miles/hr Wind Velocity.

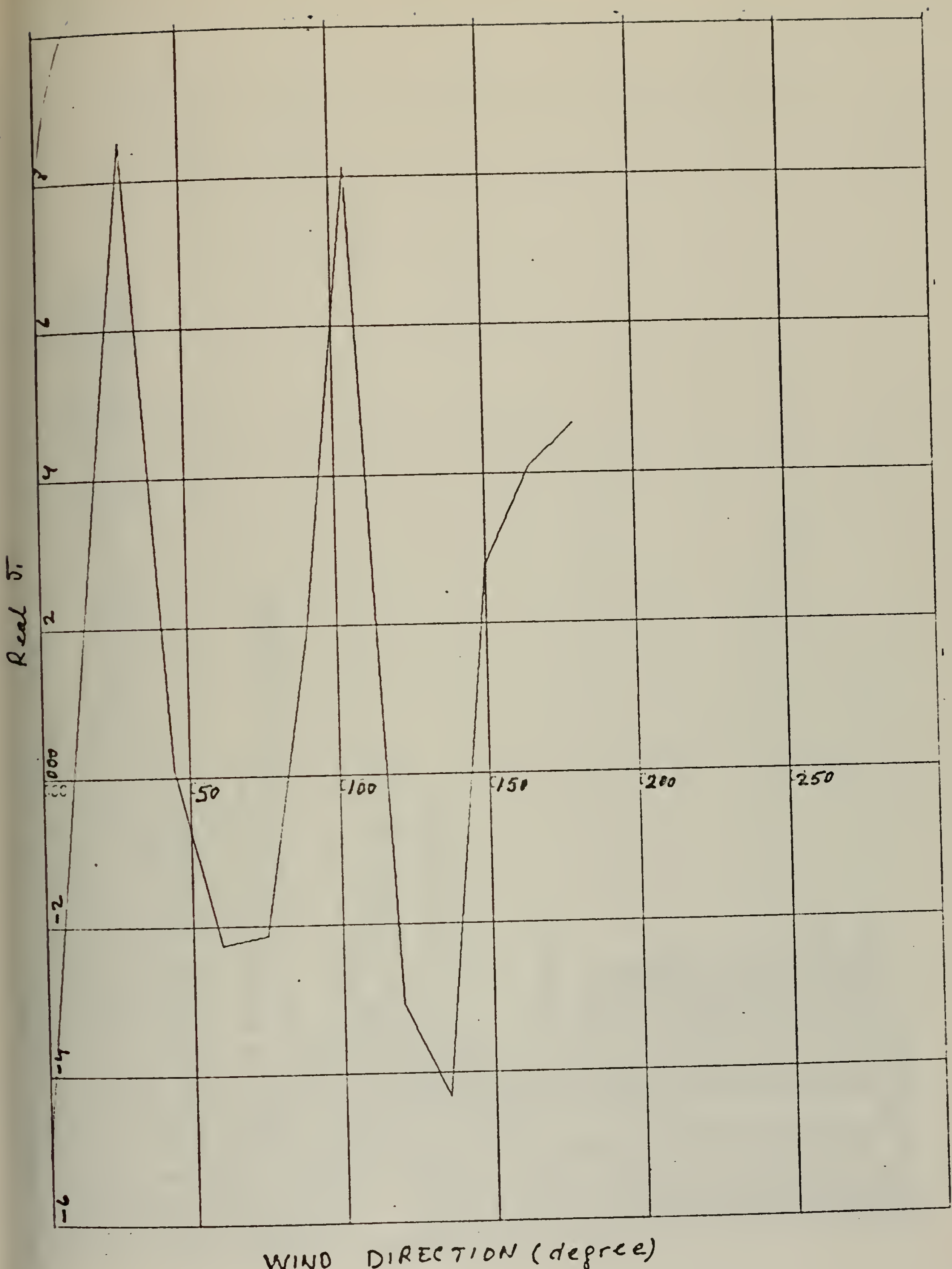


Figure 49. Real σ_1 Versus Wind Direction for 9 miles/hr Wind Velocity.

```
*** PROGRAM 1 ***
*      SHIP MOTION ON THE HORIZONTAL PLANE
*THREE DEGREE OF FREEDOM EQ.OF MOTION
*ARE PLACED IN THE FOLLOWING FORM
*(ABC.S2-AAD.S-BFA)A+(000.S2-BGC.S-000)B+(000.S2-BGD.S+000)C=AAE.D
*(000.S2-BGE.S-BFD)A+(ABD.S2-BGF.S+000)B+(000.S2-BGG.S+000)C=AAC.D
*(000.S2-CAA.S-BFG)A+(000.S2-BGA.S+000)B+(ABE.S2-CAB.S+000)C=CAC.D
*A,B,C ARE THE VARIABLES WHICH DEFINE YAW RATE , SHIP SPEED(ALONG-
*THE Y AND X AXIS) RESPECTIVELY
*D IS THE DEFLECTION OF RUDDER
TERMINAL
* SECTION 1
*WIND VELOCITY RELATIVE TO EARTH
PARAM ZYZ=1.
*DIRECTION OF WIND RELATIVE TO EARTH (DEGREE)
CONSTANT ZZZ=0.
CONSTANT QQO=0.5
*   YAW RATE GAIN CONSTANT IN A.C.S
CONSTANT QOQ=1.45
*   YAW GAIN CONSTANT IN A.C.S
*   TIME CONSTANT OF RUDDER IN C.S
CONSTANT QQQ=0.1
*HEADING ANGLE OF SHIP (DEGREE)
CONSTANT ZZY=0.
*DRIFT (DEGREE)
CONSTANT YZZ=2.
*SHIP SPEED
CONSTANT YYZ=1.
*RUDDER DEFL. (DEGREE)
CONSTANT ZDD=10.
CONSTANT AAB=0.089
CONSTANT AAC=0.0506
CONSTANT AAD=-0.071
CONSTANT AAE=0.0252
CONSTANT AAF=-0.0952
CONSTANT AAG=-0.305
CONSTANT ABA=0.0046
CONSTANT ACA=0.017
CONSTANT ADA=0.056
CONSTANT AEA=-0.015
CONSTANT ABC=0.00115
CONSTANT ABD=0.018
CONSTANT ABE=0.0095
CONSTANT ABF=-0.00001
CONSTANT DDD=0.018
CONSTANT DDE=-0.00041
*SHIP LENGTH (FEET)
CONSTANT ACC=5
```

```
*DISPLACEMENT (POUND)
CONSTANT ACD=36.2
*DENSITY OF WATER (KG/MT3)
CONSTANT ACE=1000.
*DENSITY OF AIR (KG/MT3)
CONSTANT ACF=1.29
CONSTANT 000=0.
*SECTION 2 THIS SECTION INCLUDES FOUR PARTS
*  (A)  PARAMETER CALCULATION OF THREE DEGREE FREEDOM EQ.
*       BY USING EQ. 6 THRU 20
*  (B)  CALCULATION FOR THE COEFF. OF NUM. , OF DENOM. OF
*       TR. FUNC. (YAW,SWAY) FROM THREE DEGRE  OF FREEDOM EQ.
*       BY USING CRAMERS RULE
*  (C)  CHARACTERISTIC EQ. FOR THE PARAMETER PLANE
*  (D)  POLYNOM FOR THE EIGEN VALUES
      D=ZDD/57.3
* PART  A
      BCA=(ACD*70.63)/(ACE*2.204623*(ACC**3))
      BBC=ACF/ACE
      BCB=YYZ*COS(YZZ)
      BCC=BCB/YYZ
      BCD=ZZZ+ZZY
      BCE=BCB+ZYZ*COS(BCD)
      BCF=YYZ*SIN(YZZ)
      BCG=BCF-ZYZ*SIN(BCD)
      EEE=BCE**2+BCG**2
      BDA=SQRT(EEE)
      BDB=BDA/YYZ
      BDC=BCE/YYZ
      BDE=BCG/BCE
      BDG=ATAN(BDE)
      BBA=BBC*BDB**2*(ABA/2*SIN(2*BDG)+ACA*SIN(BDG))
      BEA=BBC*BDB**2*SIN(BDG)*ADA
      BEB=BBC*BDB**2*AEA*COS(BDG)
      BEC=(BEA*AAE-BBA*AAC)/(AAF*AAC-AAG*AAE)
      BED=(BBA*AAG-BEA*AAF)/(AAF*AAC-AAG*AAE)
      BEE=ABA*COS(2*BDG)+ACA*COS(BDG)
      BEF=BDB**2-BDC
      BEG=ABA/2*SIN(2*BDG)+ACA*SIN(BDG)
      BFA=BBC*(BEE*BEF-2*BDB*SIN(BDG)*BEG)
      BFB=-BBC*BDB*(BEE*COS(BDG)+2*BEG*SIN(BDG))
      BFC=BBC*BDB*(-BEE*SIN(BDG)+2*BEG*COS(BDG))
      BFD=BBC*ADA*(COS(BDG)*BEF-2*BDB*(0.5-0.5*COS(2*BDG)))
      BFE=-BCC*BDB*ADA*(1.5-0.5*COS(2*BDG))
      BFF=BBC*BDB*ADA*COS(BDG)*SIN(BDG)
      BFG=-BBC*AEA*SIN(BDG)*(BDB**2+BDC)
      BGA=-BBC*AEA*BDB*COS(BDG)*SIN(BDG)
      BGB=BBC*AEA*BDB*(1.5+0.5*COS(2*BDG))
```

```
      BGC=AAF+BFB
      BGD=AAF*BEC+2*AAE*BED+BFC
      BGE=AAB-BCA
      BGF=AAG+BFE
      BGG=AAG*BEC+2*AAC*BED+BFF
      CAA=DDD+BEC
      CAB=2*DDE+2*ABF*BED**2+BGB
      CAC=2*ABF*BED
      UQQ=1/QQQ
*  PART  B
      VVA=AAE*ABD*ABE
      VVB=AAC*BCC*ABE-AAE*ABD*CAB-AAE*BGF*ABE
      VVC=VVB+CAC*BGD*ABD
      VVD=AAC*BGD*BGA-AAE*BGG*BGA+AAE*BGF*CAB
      VVF=VVD-AAC*BGC*CAB-CAC*BGD*BGF+CAC*BGC*BGG
      SSA=ABC*ABD*ABE
      SSB=-ABC*ABD*CAB-ABC*BGF*ABE-AAD*ABD*ABE
      SSC=AAD*ABD*CAB-ABC*BGG*BGA+ABC*BGF*CAB
      SSD=+AAD*BGF*ABE-BFA*ABD*ABE-BGE*BGC*ABE
      SSE=SSC+SSD-BGD*ABD*CAA
      SSF=+AAD*BGG*BGA-AAD*BGF*CAB+BFA*ABD*CAB
      SSG=+BFA*BGF*ABE-BGE*BGD*BGA+BGE*BGC*CAB
      SAS=CAA*BGF*BGD-BFD*BGC*ABE-CAA*BGG*BGC
      SBS=SSF+SSG+SAS-BFG*BGD*ABD
      SCS=+BFA*BGG*BGA-BFA*BGF*CAB-BFD*BGD*BGA
      SES=+BFD*BGC*CAB+BFG*BGF*BGD-BGC*BGG*BFG+SCS
      WWA=ABC*AAC*ABE
      WWC=-ABC*AAC*CAB+ABC*BGG*CAC-AAD*AAC*ABE+BGE*AAE*ABE
      WW1=AAD*AAC*CAB-AAD*BGG*CAC-BFA*AAC*ABE-BGE*AAE*CAB
      WWF=WW1+BGE*BGD*CAC+BFD*AAE*ABE+CAA*AAE*BGG-CAA*BGD*AAC
      WV1=BFA*AAC*CAB-BFA*BGG*CAC-BFD*AAE*CAB+BFD*BGD*CAC
      WVB=WV1+BFG*AAE*BGG-BFG*BGD*AAC
* PART  C
      Z10=SSB+QQQ*SSA
      Z11=SSE+QQQ*SSB
      Z12=QQQ*VVA
      Z13=SBS+QQQ*SSE
      Z14=QQQ*VVC
      Z15=SES+QQQ*SBS
      Z16=QQQ*VVF
      Z17=QQQ*SES
* PART  D
      Z1=ABC*ABD*ABE*QQQ
      Z2=ABC*ABD*ABE-ABC*ABD*CAB*QQQ-ABC*BGF*ABE*QQQ-AAD*ABD*ABE*QQQ
      S1=-ABC*ABD*CAB-ABC*BGF*ABE+ABC*BGF*CAB*QQQ
      S2=S1-AAD*ABD*ABE-BGD*ABD*CAA*QQQ
      Z3=S2+AAD*ABD*CAB*QQQ+AAD*BGF*ABE*QQQ+BFA*ABD*ABE*QQQ
      S3=ABC*BGF*CAB-AAD*ABD*CAB+AAD*BGF*ABE-AAD*BGF*CAB*QQQ
```

```
      S4=S3+BFA*ABD*ABE-BFA*ABD*CAB*QQQ-BFA*BGF*ABE*QQQ-BGD*ABD*CAA
      Z4=S4+BGD*BGF*CAA*QQQ-BGD*ABD*BFG*QQQ
      S5=-AAD*BCF*CAB-BFA*ABD*CAB-BFA*BGF*ABE
      S6=S5-BFA*BGF*CAB*QQQ-BGC*BGG*CAC*QQQ+AAE*BGG*BGA*QQQ
      Z5=S6+BGD*BGF*CAA-BGD*ABD*BFG+BGD*BGF*BFG*QQQ
      Z6=BFA*BGF*CAB-BGC*BGG*CAC*QQQ+AAE*BGG*BGA*QQQ+BGD*BGF*BFG
* SECTION 3  OUTPUT
      WRITE(6,550)
      WRITE(6,575)
      WRITE(6,150)VVA,VVC,VVF
      WRITE(6,580)
      WRITE(6,150)SSA,SSB,SSE,SBS,SES
      WRITE(6,600)
      WRITE(6,625)
      WRITE(6,150)WWA,WWC,WWF,WVB
      WRITE(6,650)
      WRITE(6,150)SSA,SSB,SSE,SBS,SES
      WRITE(6,670)
      WRITE(6,51)
      WRITE(6,680)
      WRITE(6,150)OOO,OOO,OOO,Z12,Z14,Z16
      WRITE(6,52)
      WRITE(6,680)
      WRITE(6,150)OOO,OOO,Z12,Z14,Z16,OOO
      WRITE(6,53)
      WRITE(6,680)
      WRITE(6,150)SSA,Z10,Z11,Z13,Z15,Z17
      WRITE(6,700)
      WRITE(6,680)
      WRITE(6,150)Z1,Z2,Z3,Z4,Z5,Z6
   51 FORMAT('0',' A ')
   52 FORMAT('0',' B ')
   53 FORMAT('0',' CONSTANT   ')
  150 FORMAT('0',6F12.9)
  550 FORMAT('0','  TR. FUNC. OF YAW (NUM. AND DENOM.)  ')
  575 FORMAT('0','  SECOND PWR. THRU ZERO PWR.  ')
  580 FORMAT('0','  FOURTH PWR. THRU ZERO PWR.  ')
  600 FORMAT('0','  TR. FUNC. OF SWY (NUM. AND DENOM.)  ')
  625 FORMAT('0','  THIRD  PWR. THRU ZERO PWR.  ')
  650 FORMAT('0','  SIXTH  PWR. THRU ZERO PWR.  ')
  670 FORMAT('0','  CHARAC. EQ. FOR THE PARAMETER PL.  ')
  680 FORMAT('0','  FIFTH  PWR. THRU ZERO PWR.  ')
  700 FORMAT('0','  POLYNOM FOR THE EIGEN VALUES.  ')
TIMER  FINTIM=0.4
END
STOP
ENDJOB
```

```
***PROGRAM 2 ***
*      SHIP MOTION ON THE HORIZONTAL PLANE
*THREE DEGREE OF FREEDOM EQ.OF MOTION
*ARE PLACED IN THE FOLLOWING FORM
*(ABC.S2-AAD.S-BFA)A+(000.S2-BGC.S-000)B+(000.S2-BGD.S+000)C=AAE.D
*(000.S2-BGE.S-BFD)A+(ABD.S2-BGF.S+000)B+(000.S2-BGG.S+000)C=AAC.D
*(000.S2-CAA.S-BFG)A+(000.S2-BGA.S+000)B+(ABE.S2-CAB.S+000)C=CAC.D
*A,B,C ARE THE VARIABLES WHICH DEFINE YAW RATE , SHIP SPEED(ALONG-
*THE Y AND X AXIS) RESPECTIVELY
*D IS THE DEFLECTION OF RUDDER
* SECTION 1
*WIND VELOCITY RELATIVE TO EARTH
PARAM ZYZ=7.
*DIRECTION OF WIND RELATIVE TO EARTH (DEGREE)
PARAM ZZZ=90.
*  YAW RATE GAIN CONSTANT IN A.C.S
PARAM QQO=1.364
*  YAW GAIN CONSTANT IN A.C.S
PARAM QQQ=1.972
*  TIME CONSTANT OF RUDDER IN C.S
CONSTANT QQQ=0.1
*HEADING ANGLE OF SHIP (DEGREE)
CONSTANT ZZY=0.
*DRIFT (DEGREE)
CONSTANT YZZ=2.
*SHIP SPEED
CONSTANT YYZ=1.
*RUDDER DEFL. (DEGREE)
CONSTANT ZDD=15
CONSTANT AAB=0.089
CONSTANT AAC=0.0506
CONSTANT AAD=-0.071
CONSTANT AAE=0.0252
CONSTANT AAF=-0.0952
CONSTANT AAG=-0.305
CONSTANT ABA=0.0046
CONSTANT ACA=0.017
CONSTANT ADA=0.056
CONSTANT AEA=-0.015
CONSTANT ABC=0.00115
CONSTANT ABD=0.018
CONSTANT ABE=0.0095
CONSTANT DDD=0.018
CONSTANT DDE=-0.00041
CONSTANT ABF=-0.00000001
*SHIP LENGTH (FEET)
CONSTANT ACC=5
*DISPLACEMENT (POUND)
```

```
CONSTANT ACD=36.2
*DENSITY OF WATER (KG/MT3)
CONSTANT ACE=1000.
*DENSITY OF AIR (KG/MT3)
CONSTANT ACF=1.29
*SECTION 2  PARAMETER CALCULATION OF THREE DEGREE FREEDOM EQ.
*        BY USING EQ. 6 THRU 20
      D=ZDD/57.3
      OQQ=1./QQQ
      BCA=(ACD*70.63)/(ACE*2.204623*(ACC**3))
      BBC=ACF/ACE
      BCB=YYZ*COS(YZZ)
      BCC=BCB/YYZ
      BCD=ZZZ+ZZY
      BCE=BCB+ZYZ*COS(BCD)
      BCF=YYZ*SIN(YZZ)
      BCG=BCF-ZYZ*SIN(BCD)
      EEE=BCE**2+BCG**2
      BDA=SQRT(EEE)
      BDB=BDA/YYZ
      BDC=BCE/YYZ
      BDE=BCG/BCE
      BDG=ATAN(BDE)
      BBA=BBC*BDB**2*(ABA/2*SIN(2*BDG)+ACA*SIN(BDG))
      BEA=BBC*BDB**2*SIN(BDG)*ADA
      BEB=BBC*BDB**2*AEA*COS(BDG)
      BEC=(BEA*AAE-BBA*AAC)/(AAF*AAC-AAG*AAE)
      BED=(BBA*AAG-BEA*AAF)/(AAF*AAC-AAG*AAE)
      BEE=ABA*COS(2*BDG)+ACA*COS(BDG)
      BEF=BDB**2-BDC
      BEG=ABA/2*SIN(2*BDG)+ACA*SIN(BDG)
      BFA=BBC*(BEE*BEF-2*BDB*SIN(BDG)*BEG)
      BFB=-BBC*BDB*(BEE*COS(BDG)+2*BEG*SIN(BDG))
      BFC=BBC*BDB*(-BEE*SIN(BDG)+2*BEG*COS(BDG))
      BFD=BBC*ADA*(COS(BDG)*BEF-2*BDB*(0.5-0.5*COS(2*BDG)))
      BFE=-BCC*BDB*ADA*(1.5-0.5*COS(2*BDG))
      BFF=BBC*BDB*ADA*COS(BDG)*SIN(BDG)
      BFG=-BBC*AEA*SIN(BDG)*(BDB**2+BDC)
      BGA=-BBC*AEA*BDB*COS(BDG)*SIN(BDG)
      BGB=BBC*AEA*BDB*(1.5+0.5*COS(2*BDG))
      BGC=AAF+BFB
      BGD=AAF*BEC+2*AAE*BED+BFC
      BGE=AAB-BCA
      BGF=AAG+BFE
      BGG=AAG*BEC+2*AAC*BED+BFF
      CAA=DDD+BEC
      CAB=2*DDE+2*ABF*BED**2+BGB
      CAC=2*ABF*BED
```

```
*SECTION 3 -PHYSICAL DEFINITIONS
      YAW=A
      SWAY=K
      SURGE=R
*SECTION 4 -PROGRAMMED SIMULATION
      KDOT=CDOT*SIN(YAW)+BDOT*COS(YAW)
      K=INTGRL(0.,KDOT)
      RDOT=CDOT*COS(YAW)-BDOT*SIN(YAW)
      R=INTGRL(0.,RDOT)
      ADDOT=(COFAA*I1)/DEL
      ADOT=INTGRL(0.,ADDOT)
      A=INTGRL(0.,ADOT)
      BDDOT=(COFBB*I2)/DEL
      BDOT=INTGRL(0.01,BDDOT)
      B=INIGRL(0.,BDOT)
      CDDOT=(COFCC*I3)/DEL
      CDOT=INTGRL(0.4,CDDOT)
      C=INTGRL(0.,CDOT)
      COFAA=ABD*ABE
      COFBB=ABC*ABE
      COFCC=ABC*ABD
      I1=AAD*ADOT+BFA*A+BGC*BDOT+BGD*CDOT+AAE*D
      I2=BGE*ADOT+BFD*A+BGF*BDOT+BGG*CDOT+AAC*D
      I3=CAA*ADOT+BFG*A+BGA*BDOT+CAB*CDOT+CAC*D
      DEL=ABC*ABD*ABE
*SECTION 5 -OUTPUT CHARACTERISTICS
PRTPLOT YAW
PRTPLOT SWAY
PREPAR TIME,YAW
PREPAR TIME,SWAY
TIMER  FINTIM=300.,DELT=0.1,PRDEL=0.1
END
STOP
ENDJOB
```

```
*** PROGRAM 3 ***
*       SHIP MOTION ON THE HORIZONTAL PLANE
*THREE DEGREE OF FREEDOM EQ.OF MOTION
*ARE PLACED IN THE FOLLOWING FORM
*(ABC.S2-AAD.S-BFA)A+(000.S2-BGC.S-000)B+(000.S2-BGD.S+000)C=AAE.D
*(000.S2-BGE.S-BFD)A+(ABD.S2-BGF.S+000)B+(000.S2-BGG.S+000)C=AAC.D
*(000.S2-CAA.S-BFG)A+(000.S2-BGA.S+000)B+(ABE.S2-CAB.S+000)C=CAC.D
*A,B,C ARE THE VARIABLES WHICH DEFINE YAW RATE , SHIP SPEED(ALONG-
*THE Y AND X AXIS) RESPECTIVELY
*D IS THE DEFLECTION OF RUDDER
*  FOR THE AUTOMATICALLY STEERED SHIP (ZDD=0 )
*SECTION 4 -         CONTROL LOOP PROGRAM SIMULATION
      KDOT=CDOT*SIN(YAW)+BDOT*COS(YAW)
      K=INTGRL(0.,KDOT)
      RDOT=CDOT*COS(YAW)-BDOT*SIN(YAW)
      R=INTGRL(0.,RDOT)
      ADDDOT=(COFAA*I1)/DEL
      ADDOT=INTGRL(0.,ADDDOT)
      ADOT=INTGRL(0.,ADDOT)
      A=INTGRL(0.,ADOT)
      BDDDOT=(COFBB*I2)/DEL
      BDDOT=INTGRL(0.,BDDDOT)
      BDOT=INTGRL(0.,BDDOT)
      B=INTGRL(0.,BDOT)
      CDDDOT=(COFCC*I3)/DEL
      CDDOT=INTGRL(0.,CDDDOT)
      C=INTGRL(0.,CDOT)
      COFAA=ABE*ABD
      COFBB=ABC*ABE
      COFCC=ABC*ABD
      V1=(AAD-OQQ*ABC)*ADDOT+(BFA+OQQ*AAD-AAE*OQQ*QQO)*ADOT
      V2=V1+(OQQ*BFA-AAF*OQQ*QOQ)*A+BGC*BDDOT+BGC*OQQ*BDOT
      I1=V2+BGD*CDDOT+BGD*OQQ*CDOT
      V3=BGE*ADDOT+(BFD+BGE*OQQ-AAC*OQQ*QQO)*ADOT
      V4=V3+(BFD*OQQ-AAC*OQQ*QCQ)*A+(BGF-ABD*OQQ)*BDDOT
      I2=V4+BGF*OQQ*BDOT+BGG*CDDOT+BGG*OQQ*CDOT
      V5=CAA*ADDOT+(BFG+CAA*OQQ-CAC*OQQ*QQO)*ADOT
      V6=V5+(BFG*OQQ-CAC*OQQ*QOQ)*A+BGA*BDDOT+BGA*OQQ*BDOT
      I3=V6+(CAB-ABE*OQQ)*CDDOT+CAB*OQQ*CDOT
      DEL=ABC*ABD*ABE
*SECTION 5 -OUTPUT CHARACTERISTICS
PRTPLOT YAW
PRTPLOT SWAY
PREPAR TIME,YAW
PREPAR SWAY
TIMER  FINTIM=90.,DELT=0.02,PRDEL=0.2
END
STOP
```

```
C****                   PARAMETER PLANE USING MATRIX METHODS                  ****
C****                               PARAM M                                   ****
C                                                                                *
C     PROGRAM PARAM M CALCULATES AND PLOTS CURVES ON THE TWO DIMENSIONAL         *
C     PARAMETER PLANE OF EITHER:                                                 *
C        1) CONSTANT SIGMA(FNC OMEGA) AND/OR CONSTANT OMEGA(FNC SIGMA);          *
C        2) CONSTANT ZETA(FNC OMEGA(N)) AND/OR CONSTANT OMEGA(N)(FNC ZETA);      *
C        3) SIGMA (REAL ROOT) LINES;                                             *
C        4) A COMBINATION OF 1 AND 3 OR 2 AND 3 ABOVE;                           *
C     FOR CHARACTERISTIC EQUATIONS OF THE TYPE                                   *
C        A(0) + A(1)*S + - - - + A(N-2)*S**(N-2) + A(N-1)*S**(N-1) + S**N        *
C     WHERE THE A COEFFICIENTS ARE OF THE LINEAR FORM                            *
C        B*ALPHA + C*BETA + D  WITH B,C, AND D CONSTANTS.                        *
C     IN ADDITION, PROVISIONS ARE INCLUDED IN THE PROGRAM TO COMPUTE THE         *
C     ROOTS OF THE REDUCED CHARACTERISTIC EQUATION FOR THE GIVEN SIGMA           *
C     AND OMEGA OR ZETA AND OMEGA(N) TO ALLOW FOR SHADING (STABILITY) OF         *
C     THE PARAMETER PLANE.                                                       *
C                                                                                *
C************* DEFINITION OF SYMBOLS USED FOR INPUT DATA *************************
C                                                                                *
C     NT= TYPE OF OUTPUT CURVES DESIRED                                          *
C         1=CONSTANT SIGMA(FNC OMEGA) AND/OR CONSTANT OMEGA(FNC SIGMA)           *
C         2=CONSTANT ZETA(FNC OMEGA(N)) AND/OR CONSTANT OMEGA(N)(FNC ZETA)       *
C         3=SIGMA(REAL ROOT) LINES                                               *
C         4=COMBINATION OF 1 AND 3 ABOVE                                         *
C         5=COMBINATION OF 2 AND 3 ABOVE                                         *
C           NOTE: ALL CURVES ARE PLOTTED ON PLAIN PAPER.  IF A 1 INCH GRID       *
C                 IS DESIRED REPLACE ABOVE NT VALUES WITH 11,12,13,14,15         *
C     NO= ORDER OF CHARACTERISTIC EQUATION (11>NO>1)                             *
C     ND= NO. OF DECADES SPANNED BY OMEGA OR OMEGA(N)IF NT=1,2,4,5               *
C     NL= LARGEST SIGMA/ZETA FOR CONSTANT OMEGA/OMEGA(N) CURVES NT=1,2,4,5       *
C     NZ= NO. OF CONSTANT: SIGMA CURVES IF NT=1,4; ZETA CURVES IF NT=2,5         *
C     NW= NO. OF CONSTANT: OMEGA CURVES IF NT=1,4;OMEGA(N) CURVES IF NT=2,5      *
C     NS= NO. OF SIGMA(REAL ROOT) LINES IF NT=3,4,5                              *
C     IP= PRINTED OUTPUT DESIRED                                                 *
C         1=FIRST PLOT PT EACH CURVE PLUS SUB PLOT PTS IF STABILITY CHANGES      *
C         2=OUTPUT EVERY TENTH PLOTTED POINT PLUS 1 ABOVE                        *
C         3=OUTPUT EVERY PLOTTED PT (MAX OF 300+100*(NO-2) LINES PER CURVE)      *
C         4=OUTPUT EVERY ITERATION STEP (300+100*(NO-2)LINES PER CURVE)          *
C         5=OUTPUT EVERY ITERATION WITHOUT ROOTS (300 LINES PER CURVE)           *
C         6=OUTPUT EVERY TENTH ITERATION WITHOUT ROOTS (30 LINES PER CURVE)      *
C         7=NO PRINTED OUTPUT                                                    *
C     IX= DISTANCE IN INCHES OF THE X-AXIS FROM THE BOTTOM OF THE GRAPH          *
C     IY= DISTANCE IN INCHES OF THE Y-AXIS FROM LEFT SIDE OF THE GRAPH           *
C     WS= START VALUE OF OMEGA/OMEGA(N) FOR CONSTANT SIGMA/ZETA CURVES           *
C     XS= X-SCALE IN UNITS PER INCH (CANNOT BE ZERO)                             *
C     YS= Y-SCALE IN UNITS PER INCH (CANNOT BE ZERO)                             *
C     LZ= LABELS FOR CONSTANT: SIGMA(NI=1,4) OR ZETA (NI=2,5) CURVES             *
```

```
C     LW= LABELS FOR CONSTANT: OMEGA(NI=1,4) OR OMEGA(N)(NI=2,5)CURVES          * C49
C     LS= LABELS FOR SIGMA (REAL ROOT) LINES                                    * C50
C                                                                               * C51
C************************ DATA CARDS REQUIRED ********************************* C52
C                                                                               * C53
C     CARD 1  FIRST LINE OF GRAPH TITLE (COLUMNS 1-48)                          * C54
C     CARD 2  SECOND LINE OF GRAPH TITLE (COLUMNS 1-48)                         * C55
C     CARD 3  NI,NO,NP,NY,NZ,NW,NS,IP,IX,IY,WS,XS,YS (10I5,3E10.5 FORMAT)       * C56
C     CARD 4  LZ (20A4 FORMAT), USE BLANK CARD IF NI=3                          * C57
C     CARD 5  LW (20A4 FORMAT), USE BLANK CARD IF NI=3                          * C58
C     CARD 6  LS (20A4 FORMAT), USE BLANK CARD IF NI=1,2                        * C59
C     CARD 7  CORRESPONDING VALUES OF CARD 4 (8E10.5 FORMAT)                    * C60
C     CARD 8  CORRESPONDING VALUES OF CARD 5 (8E10.5 FORMAT)                    * C61
C     CARD 9  CORRESPONDING VALUES OF CARD 6 (8E10.5 FORMAT)                    * C62
C     CARD 10 CONSTANT COEFFICIENTS IN ASCENDING ORDER (8E10.5 FORMAT)          * C63
C     CARD 11 ALPHA COEFFICIENTS IN ASCENDING ORDER (8E10.5 FORMAT)             * C64
C     CARD 12 BETA  COEFFICIENTS IN ASCENDING ORDER (8E10.5 FORMAT)             * C65
C                                                                               * C66
C*******************************************************************************
C************************** SAMPLE DATA DECK ********************************** C67
C                            (SCALE COMPRESSED)                                 * C68
C                                                                               * C69
C   LINE |   |  |         |         |         |         |         |         |   * C70
C        1   5  10        20        30        40        50        60        70        80  * C71
C                                                                               * C72
CARD 1  DUBAC  PARAMETER PLANE CURVES                                           * C73
CARD 2  S4+(20+A)S3+(1700+20A)S2+(1700A+B)S+25B                                 * C74
CARD 3      2    4    2    1    6    5    0    1    0    0 1.0       3.0   .3   E+04 * C75
CARD 4  Z0.0Z0.2Z0.4Z0.6Z0.8Z1.0                                                * C76
CARD 5  WN 5WN10WN20WN30WN40                                                    * C77
CARD 6  (BLANK CARD)                                                            * C78
CARD 7      0.0       0.2       0.4       0.6       0.8       1.0               * C79
CARD 8      5.0      10.0      20.0      30.0      40.0                         * C80
CARD 9                                                                          * C81
CARD 10     0.0       0.0    .17 E+04 20.0                                      * C82
CARD 11     0.0    .17 E+04 20.0       1.0                                      * C83
CARD 12    25.0       1.0                                                       * C84
C                                                                               * C85
C****                   LIMITATIONS AND ERROR READOUTS                       **** C86
C                                                                               * C87
C     LIMITATIONS:                                                              * C88
C        1. ORDER OF CHARACTERISTIC EQUATION LIMITED TO 10.  CAN BE             * C89
C           INCREASED TO N BY CHANGING DIMENSIONS OF AK TO N,N; CK,BK,DK,       * C90
C           SIGP TO N;  COEF,COF TO N-1;  ROOTR,ROOTI TO N-2; AND AA TO NXN     * C91
C        2. NUMBER OF CURVES LIMITED TO 20 PER TYPE (MAXIMUM OF SIXTY).         * C92
C           CAN BE INCRESED TO M BY CHANGING DIMENSIONS OF LZ,LW,LS,SZ,WWN,     * C93
C           SIG TO M.                                                           * C94
C                                                                               * C95
```

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C     ERROR READOUTS:                                                    * C96
C        1: POSSIBLE SINGULAR POINT                                      * C97
C           MEANING: MATRIX BEING INVERTED EITHER SINGULAR OR NEARLY SO: * C98
C                    COEF OF REDUCED CHAR. EQN.:                         * C99
C        2: POLRT ERROR:                                                 *C100
C           MEANING: SUBROUTINE POLRT UNABLE TO DETERMINE ROOTS WITH 500 *C101
C                    ITERATIONS ON 5 STARTING VALUES.  COEFFICIENTS ARE  *C102
C                    PRINTED OUT FOR CONVENIENCE.                        *C103
C                                                                       ****C104
C****                       TEMPORARY RESTRICTION                        *C105
C                                                                        *C106
C     DUE TO SOME AS YET UNDETERMINED QUIRK, AN ERROR IN THE POLRT SUB-  *C107
C     ROUTINE PERIODICALLY CAUSES AN ERROR INTERRUPT WHICH IN THE MVT MODE *C108
C     OF OPERATION ON THE IBM 360 SELF CORRECTS ITSELF.  UNTIL SUCH TIME *C109
C     AS THIS DESCREPANCY CAN BE ELIMINATED, THIS PROGRAM MUST BE RUN ON MVT*C110
C     ONLY.  IF ANY SUCH INTERRUPTS OCCUR, THE FOLLOWING MESSAGE WILL    *C111
C     APPEAR AT THE END OF THE PRINT OUT:                                *C112
C           SUMMARY OF ERRORS FOR THIS JOB   ERROR NUMBER  NUMBER OF ERRORS *C113
C                                              209          "NO. ERRORS"  C114
C                                                                         0002
      REAL*8 ITITLE(12)                                                   0003
      REAL LABEL/4H    /,GG(10)/1.0076,1.016,1.0245,1.0312,1.0394,        0004
     *1.0483,1.0568,1.0633,1.071,1.078/, AK(10,10),LZ(20),LW(20),         0005
     *LS(20),SZ(20),WWN(20),SIG(20),CK(10),BK(10),DK(10),AA(100),         0006
     *COEF(9),COF(9),ROOTR(8),ROOTI(8),ALFA(300),BETA(300),SIGP(10)       TEMP
      CALL ERRSET (207,256,-1,1,0,209)                                    0007
      WRITE(6,500)                                                        0008
      READ(5,400)(ITITLE(I),I=1,12)                                       0009
      WRITE(6,401)(ITITLE(I),I=1,12)                                      0010
      READ(5,402) NT,NO,ND,NL,NZ,NW,NS,IP,IX,IY,WN,XS,YS                  0011
      WRITE(6,501)                                                        0012
      WRITE(6,411) NT,NO,ND,NL,NZ,NW,NS,IP,IX,IY,WN,XS,YS                 0013
      IG=0                                                                0014
      IF(NT.LE.5) GO TO 200                                               0015
      IG=1                                                                0016
      NT=NT-10                                                            0017
  200 M1=NO+1                                                             0018
      M2=NO-2                                                             0019
      M3=NO-1                                                             0020
      READ(5,403)(LZ(I),I=1,NZ)                                           0021
      WRITE(6,502)                                                        0022
      WRITE(6,404)(LZ(I),I=1,NZ)                                          0023
      READ(5,403)(LW(I),I=1,NW)                                           0024
      WRITE(6,503)                                                        0025
      WRITE(6,404)(LW(I),I=1,NW)                                          0026
      READ(5,403)(LS(I),I=1,NS)                                           0027
      WRITE(6,504)                                                        0028
      WRITE(6,404)(LS(I),I=1,NS)                                          0029
      READ(5,405)(SZ(I),I=1,NZ)
```

```
      WRITE(6,505)
      WRITE(6,406)(SZ(I),I=1,NZ)
      READ(5,405)(WWN(I),I=1,NW)
      WRITE(6,506)
      WRITE(6,406)(WWN(I),I=1,NW)
      READ(5,405)(SIG(I),I=1,NS)
      WRITE(6,507)
      WRITE(6,406)(SIG(I),I=1,NS)
      READ(5,405)(CK(M1-I),I=1,NO)
      WRITE(6,508)
      WRITE(6,406)(CK(M1-I),I=1,NO)
      READ(5,405) (AK(M1-I,1),I=1,NO)
      WRITE(6,509)
      WRITE(6,406)(AK(M1-I,1),I=1,NO)
      READ(5,405) (AK(M1-I,2),I=1,NO)
      WRITE(6,510)
      WRITE(6,406)(AK(M1-I,2),I=1,NO)
      MOD=1
      XLIMP=(9.2-IY)*XS
      YLIMP=(15.2-IX)*YS
      XLIMN=-IY*XS
      YLIMN=-IX*YS
      IF(NO.EQ.2) GO TO 202
      DO 101 I=1,NO
      DO 100 J=1,M2
  100 AK(I,J+2)=0.0
  101 BK(I)=0.0
      DO 102 I=1,M2
  102 AK(I,I+2)=-1.0
  202 GO TO(103,103,147,103,103),NT
  103 DO 146 JJ=1,2
      GO TO (204,205),JJ
  204 IF(NZ.EQ.0) GO TO 146
      NI=NZ
      G=GG(ND)
      WRITE(6,526)
      GO TO 206
  205 IF(NW.EQ.0) GO TO 146
      NI=NW
      DEL=NL/300.0
      WRITE(6,526)
  206 DO 145 K=1,NI
      GO TO (104,207),JJ
  104 W=WN
      SIGZ=SZ(K)
      GO TO 105
  207 SIGZ=0.0
      W=WWN(K)
```

```
105 LL=0
    LLL=0
    LLLL=0
    IPSET=0
    DO 242 L=1,300
    IPP=0
    GO TO(106,109,146,106,109),NT
106 IF(L.GT.1) GO TO 607
    GO TO (306,307),JJ
306 WRITE(6,511)
    WRITE(6,513)
    GO TO 607
307 WRITE(6,512)
    WRITE(6,513)
607 IF(NO.EQ.2) GO TO 208
    DO 107 I=2,M3
107 AK(I,I+1)=-2.0*SIGZ
    DO 108 I=3,NO
108 AK(I,I)=-(SIGZ**2 + W**2)
208 BK(1)=2.0*SIGZ
    BK(2)=SIGZ**2+W**2
    GO TO 112
109 IF(L.GT.1) GO TO 310
    GO TO (209,210),JJ
209 WRITE(6,522)
    WRITE(6,524)
    GO TO 310
210 WRITE(6,523)
    WRITE(6,524)
310 IF(NO.EQ.2) GO TO 211
    DO 110 I=2,M3
110 AK(I,I+1)=-2.0*SIGZ*W
    DO 111 I=3,NO
111 AK(I,I)=-W**2
211 BK(1)=2.0*SIGZ*W
    BK(2)=W**2
112 DO 113 I=1,NO
113 DK(I)=BK(I)-CK(I)
    II=0
    DO 115 I=1,NO
    DO 114 J=1,NO
    II=II+1
114 AA(II)=AK(J,I)
115 CONTINUE
    CALL SIMQ(AA,DK,NO,KS)
    KS=KS+1
    GO TO (117,116),KS
116 WRITE(6,407)SIGZ,W
```

```
      GO TO 240
  117 ATEMP=DK(1)
      BTEMP=DK(2)
      IF(L.GT.1) GO TO 217
      AMAX=ATEMP
      AMIN=ATEMP
      BMAX=BTEMP
      BMIN=BTEMP
  217 IF(ATEMP.GE.AMIN) GO TO 317
      AMIN=ATEMP
  317 IF(ATEMP.LE.AMAX) GO TO 417
      AMAX=ATEMP
  417 IF(BTEMP.GE.BMIN) GO TO 617
      BMIN=BTEMP
  617 IF(BTEMP.LE.BMAX) GO TO 717
      BMAX=BTEMP
  717 IF(NO.EQ.2) GO TO 218
      DO 118 I=1,M2
  118 COEF(I)=DK(M1-I)
      COEF(M3)=1.0
  218 IF(XLIMN-ATEMP)119,119,123
  119 IF(ATEMP-XLIMP)120,120,123
  120 IF(YLIMN-BTEMP)121,121,123
  121 IF(BTEMP-YLIMP) 122,122,123
  122 LL=LL+1
      LLL=LLL+1
      GO TO (125,125,125,125,138,234,140),IP
  123 GO TO (240,240,240,124,124,124,240),IP
  124 IPP=1
      GO TO (125,125,125,125,138,234,142),IP
  125 IF(NO.GT.3) GO TO 225
      IF(NO.EQ.2) GO TO 127
      ROOTR(1)=-DK(3)
      ROOTI(1)=0.0
      GO TO 127
  225 CALL POLRT(COEF,COF,M2,ROOTR,ROOTI,IER)
      IERROR=IER+1
      GO TO(127,126,126,126,126),IERROR
  126 WRITE(6,408)ATEMP,BTEMP,SIGZ,W
      WRITE(6,412) (COEF(M3+1-I),I=1,M3)
      IF(LL.GT.1) GO TO 139
      IPSET=1
      GO TO 139
  127 GO TO (128,128,138,138,138,138,142),IP
  128 ISTAB=0
      IF(NO.EQ.2) GO TO 130
      DO 130 J=1,M2
      IF(ROOTR(I)) 130,130,129
```

```
129 ISTAB=1
130 CONTINUE
    IF(LL.EQ.1.OR.IPSET.EQ.1) GO TO 231
    GO TO 131
231 ISTABL=ISTAB
131 IF(ISTABL-ISTAB)335,132,335
132 IF(LL.EQ.1.OR.IPSET.EQ.1) GO TO 138
    GO TO (140,133,138,138,138,138,142),IP
133 IF(LLL-10)140,134,134
134 LLL=0
    GO TO 138
234 IF(L.EQ.1.OR.LLLL.EQ.10) GO TO 235
    GO TO 139
235 LLLL=0
    GO TO 138
335 ISTABL=ISTAB
135 GO TO (138,136,138,138,138,138,240),IP
136 IF(LLL-10)138,137,137
137 LLL=0
138 WRITE(6,409)ATEMP,BTEMP,SIGZ,W
    IPSET=0
    GO TO (238,238,238,238,139,139,240),IP
238 IF(NO.EQ.2) GO TO 139
    WRITE(6,410)(ROOTR(I),ROOTI(I),I=1,M2)
139 IF(IPP.EQ.1)GO TO 240
140 ALFA(LL)=ATEMP
    BETA(LL)=BTEMP
240 IF(JJ.EQ.1)GO TO 141
    SIGZ=SIGZ+DEL
    GO TO 142
141 W=W*G
142 LLLL=LLLL+1
242 CONTINUE
    IF(LL.LT.2)GO TO 144
    IF(JJ.EQ.1)GO TO 143
    CALL DRAW(LL,ALFA,BETA,MOD,0,LW( K),ITITLE,XS,YS,IX,IY,2,2,
   *9,15,IG,LAST)
    WRITE(6,519) W,LL
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
    MOD=2
    GO TO 145
143 CALL DRAW(LL,ALFA,BETA,MOD,0,LZ(K),ITITLE,XS,YS,IX,IY,2,2,
   *9,15,IG,LAST)
    WRITE(6,520) SIGZ,LL
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
    MOD=2
    GO TO 145
144 GO TO (244,344),JJ
```

```
244 WRITE(6,514) SIGZ
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
    GO TO 145
344 WRITE(6,521) W
    WRITE(6,525) AMAX,BMAX,AMIN,BMIN
145 CONTINUE
146 CONTINUE
147 GO TO (165,165,148,148,148),NT
148 GO TO (248,248,248,248,248,248,249),IP
248 WRITE(6,515)
249 DO 164 K=1,NS
    DO 149 I=1,NO
149 SIGP(I)=SIG(K)**I
    ACOEF=AK(NO,1)
    BCOEF=AK(NO,2)
    CCOEF=SIGP(NO)+CK(NO)
    DO 150 I=1,M3
    ACOEF=ACOEF+AK(I,1)*SIGP(NO-I)
    BCOEF=BCOEF + AK(I,2)*SIGP(NO-I)
150 CCOEF=CCOEF+CK(I)*SIGP(NO-I)
    SLOPE=-BCOEF/ACOEF
    AINTER=-CCOEF/ACOEF
    GO TO (250,250,250,250,250,250,164),IP
250 WRITE(6,516) SIG(K),SLOPE,AINTER
    J=1
    ALFA(J)=SLOPE*YLIMN+AINTER
    IF(XLIMN-ALFA(J)) 151,151,153
151 IF(ALFA(J)-XLIMP) 152,152,153
152 BETA(J)=YLIMN
    J=J+1
153 ALFA(J)=SLOPE*YLIMP+AINTER
    IF(XLIMN-ALFA(J)) 154,154,156
154 IF(ALFA(J)-XLIMP) 155,155,156
155 BETA(J)=YLIMP
    IF(J.EQ.2) GO TO 162
    J=J+1
156 BETA(J)=(1.0/SLOPE)*XLIMN-AINTER/SLOPE
    IF(YLIMN-BETA(J)) 157,157,159
157 IF(BETA(J)-YLIMP) 158,158,159
158 ALFA(J)=XLIMN
    IF(J.EQ.2) GO TO 162
    J=J+1
159 BETA(J)=(1.0/SLOPE)*XLIMP-AINTER/SLOPE
    IF(YLIMN-BETA(J)) 160,160,163
160 IF(BETA(J)-YLIMP) 161,161,163
161 ALFA(J)=XLIMP
    IF(J.LT.2) GO TO 163
162 CALL DRAW(2,ALFA,BETA,MOD,0,LS(K),ITITLE,XS,YS,IX,IY,2,2,
```

```
      *9,15,0,LAST)
       MOD=2
       GU TO 164
   163 WRITE(6,517) SIG(K)
   164 CONTINUE
   165 IF(MOD.EQ.1) GO TO 166
       ALFA(1)=0.0
       ALFA(2)=1.0
       BETA(1)=0.0
       BETA(2)=0.0
       CALL DRAW(2,ALFA,BETA,3,0,LABEL,ITITLE,XS,YS,IX,IY,2,2,
      *9,15,0,LAST)
       GO TO 167
   166 WRITE(6,518)
   167 CONTINUE
   400 FORMAT(6A8)
   401 FORMAT(2X,6A8)
   402 FORMAT(10I5,3E10.5)
   403 FORMAT(20A4)
   404 FORMAT(2X,20(A4,2X))
   405 FORMAT(8E10.5)
   406 FORMAT(2X,8E14.5)
   407 FORMAT(T7,'POSSIBLE SINGULAR POINT',2E14.5,T64,'NO ROOTS;'
      *T73,'POINT NOT PLOTTED ON GRAPH')
   408 FORMAT(4E14.5,T73,'POLRT ERROR. COEF OF REDUCED CHAR. EQN.:')
   409 FORMAT(4E14.5)
   410 FORMAT(58X,6E12.4)
   411 FORMAT(10I5,1X,3E12.4)
   412 FORMAT(76X,4E12.4)
   500 FORMAT('1',T6,'THE INPUT DATA IS:',////)
   501 FORMAT(//,3X,2HNT,3X,2HNO,3X,2HND,3X,2HNL,3X,2HNZ,3X,2HNW,3X,
      *2HNS,3X,2HIP,3X,2HIX,3X,2HIY,T53,'START OMEGA',T68,'X-SCALE',
      *T80,'Y-SCALE',/)
   502 FORMAT(///,T3,'LABELS FOR CONSTANT SIGMA OR ZETA CURVES',/)
   503 FORMAT(///,T3,'LABELS FOR CONSTANT OMEGA OR OMEGA(N) CURVES',/)
   504 FORMAT(///,T3,'LABELS FOR SIGMA(REAL ROOT) LINES',/)
   505 FORMAT(///,T3,'VALUES FOR CONSTANT SIGMA OR ZETA CURVES',/)
   506 FORMAT(///,T3,'VALUES FOR CONSTANT OMEGA OR OMEGA(N) CURVES',/)
   507 FORMAT(///,T3,'VALUES FOR SIGMA(REAL ROOT) LINES',/)
   508 FORMAT(///,T3,'CONSTANT COEFFICIENTS IN ASCENDING ORDER',/)
   509 FORMAT(///,T3,'CONSTANT ALPHA COEFFICIENTS IN ASCENDING ORDER',/)
   510 FORMAT(///,T3,'CONSTANT BETA  COEFFICIENTS IN ASCENDING ORDER',/)
   511 FORMAT(///,T19,'CONSTANT SIGMA CURVE ',T81,'CORRESPONDING REMAININ
      *G ROOTS',/)
   512 FORMAT(///,T19,'CONSTANT OMEGA CURVE ',T81,'CORRESPONDING REMAININ
      *G ROOTS',/)
   513 FORMAT(T7,'ALPHA',T21,'BETA',T34,'SIGMA',T49,'OMEGA',T64,'REAL',
      *T76,'IMAG',T88,'REAL',T100,'IMAG',T112,'REAL',T124,'IMAG',//)
```

```
514 FORMAT(T8,'CURVE FOR CONSTANT SIGMA/ZETA =',E13.5,T53,'WAS NOT PLO
   *TTED',/,T8,'AS LESS THAN TWO POINTS WERE GENERATED WITHIN SPECIFIE
   *D GRAPH RANGE',/)
515 FORMAT('1',//,T25,'SIGMA (REAL ROOT) LINES',//)
516 FORMAT(T3,'FOR SIGMA =',E12.5,T27,': ALPHA = ',E12.5,T50,
   *'* BETA + ',E12.5,/)
517 FORMAT(T3,'REAL ROOT LINE FOR SIGMA =',E13.5,T42,'LIES OUTSIDE THE
   * SPECIFIED GRAPH RANGE',/)
518 FORMAT(////,T3,'THERE WILL BE NO OUTPUT GRAPH PLOTTED AS NONE',/,
   *T3,'OF THE POINTS GENERATED BY THE PROGRAM LIE',/,
   *T3,'WITHIN THE SPECIFIED GRAPH REGION. CHECK THE VALUES OF',/,
   *T3,'XS AND YS AS WELL AS IX AND IY ON DATA CARD 3',/)
519 FORMAT(T8,'FOR THE CURVE OF CONSTANT OMEGA/OMEGA(N) =',E13.5,
   */,I10,T12,
   *'POINTS OUT OF A POSSIBLE 300 LIE WITHIN SPECIFIED GRAPH RANGE',/)
520 FORMAT(T8,'FOR THE CURVE OF CONSTANT SIGMA/ZETA =',E13.5,
   */,I10,T12,
   *'POINTS OUT OF A POSSIBLE 300 LIE WITHIN SPECIFIED GRAPH RANGE',/)
521 FORMAT(T8,'CURVE FOR CONSTANT OMEGA/OMEGA(N) =',E13.5,T57,'WAS NOT
   * PLOTTED',/,T8,'AS LESS THAN TWO POINTS WERE GENERATED WITHIN SPEC
   *IFIED GRAPH RANGE',/)
522 FORMAT(///,T19,'CONSTANT ZETA CURVE ',T81,'CORRESPONDING REMAINING
   * ROOTS',/)
523 FORMAT(///,T17,'CONSTANT OMEGA(N) CURVE ',T81,'CORRESPONDING REMAI
   *NING ROOTS',/)
524 FORMAT(T7,'ALPHA',T21,'BETA',T35,'ZETA',T48,'OMEGA(N)',T64,'REAL',
   *T76,'IMAG',T88,'REAL',T100,'IMAG',T112,'REAL',T124,'IMAG',//)
525 FORMAT(T8,'MAXIMUM AND MINIMUM GENERATED ALPHA & BETA VALUES WERE:
   *',/,T8,'AMAX=',E10.3,T24,'BMAX=',E10.3,T40,'AMIN=',E10.3,T56,
   *'BMIN=',E10.3,/)
526 FORMAT('1')
    END
```

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STRACT

The objective of this study was to examine the relationship between forward ship motion and the effects of aerodynamic and hydrodynamic disturbances on motions in the horizontal plane. It includes course control and stability analysis for the unsteered and steered cases (manual and automatic) using several sets of operating conditions.

Hydrodynamic and aerodynamic effects are considered to be functions of hull motion, rudder (steered cases), propeller and wind effects ratio. A dimensionless mathematical model has been developed and solved with respect to ship axes.

KEY WORDS	LINK A		LINK B		LINK C	
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